DOCUMENT RESUME

ED 073 122 TM 002 364

AUTHOR Joreskog, Karl G.; van Thillo, Marielle
TITLE LISREL: A General Computer Program for Estimating a Linear Structural Equation System Involving Multiple Indicators of Unmeasured Variables.
INSTITUTION Educational Testing Service, Princeton, N.J.
REPORT NO ETS-RB-72-56
PUB DATE Dec 72
NOTE 73p.; Draft

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Bulletins; *Computer Programs; *Data Analysis; *Input Output; *Linear Programing; *Mathematical Models

ABSTRACT

A general computer program for estimating the unknown coefficients in a set of linear structural equations is described. In its most general form, the variables in the equation system may be unmeasured hypothetical constructs or latent variables, and there may be several measured variables or multiple indicators for each unmeasured variable. Also, the method allows for both errors in equations (residuals, disturbances) and errors in the observed variables (errors of measurement, observational errors) and yields estimates of the disturbance variance-covariance matrix and the measurement error variances, as well as estimates of the unknown coefficients in the structual equations, provided that all these parameters are identified. The method is so general and flexible that it is possible to handle a wide range of models. The model considered here is a generalization of the model considered by Joreskog (1973). (Author/DB)

LISREL

A GENERAL COMPUTER PROGRAM FOR ESTIMATING A LINEAR STRUCTURAL EQUATION SYSTEM INVOLVING MULTIPLE INDICATORS OF UNMEASURED VARIABLES

Karl G. Jöreskog
University of Uppsala, Sweden

and

Marielle van Thillo
Educational Testing Service

This Bulletin is a draft for interoffice circulation. Corrections and suggestions for revision are solicited. The Bulletin should not be cited as a reference without the specific permission of the authors. It is automatically superseded upon formal publication of the material.

Educational Testing Service
Princeton, New Jersey
December 1972

LISREL

A General Computer Program for Estimating a Linear Structural Equation System Involving Multiple Indicators of Unmeasured Variables

1. Introduction

We shall describe a general computer program for estimating the unknown coefficients in a set of linear structural equations. In its most general form the variables in the equation system may be unmeasured hypothetical constructs or latent variables and there may be several measured variables or multiple indicators for each unmeasured variable. Also, the method allows for both errors in equations (residuals, disturbances) and errors in the observed variables (errors of measurement, observational errors) and yields estimates of the disturbance variance-covariance matrix and the measurement error variances as well as estimates of the unknown coefficients in the structural equations, provided that all these parameters are identified. The method is so general and flexible that it is possible to handle a wide range of models, for example, path analysis models (Wright, 1934, 1954, 1960; Turner & Stevens, 1959; Duncan, 1966; Duncan, Haller & Portes, 1968; Land, 1969; Heise, 1970; Blalock, 1969, 1971; Costner, 1969; Hauser & Goldberger, 1971), econometric models (Goldberger, 1964; Malinvaud, 1970; Johnston, 1972), factor analysis and covariance structure models (Jöreskog, 1969, 1970). The model considered here is a generalization of the model considered by Jöreskog (1973).

1.1. The General Model

Consider random vectors $\eta' = (\eta_1, \eta_2, \ldots, \eta_m)$ and $\xi' = (\xi_1, \xi_2, \ldots, \xi_n)$ of true dependent and independent variables, respectively, and the following system of linear structural relations

$$B\eta = \Gamma\xi + \zeta \quad (1)$$

where $B(m \times m)$ and $\Gamma(m \times n)$ are coefficient matrices and $\zeta' = (\zeta_1, \zeta_2, \ldots, \zeta_m)$ is a random vector of residuals (errors in equations, random disturbance terms). Without loss of generality it may be assumed that $\mathcal{E}(\eta) = \mathcal{E}(\zeta) = 0$ and $\mathcal{E}(\xi) = 0$. It is furthermore assumed that ζ is uncorrelated with ξ and that B is nonsingular.

The vectors η and ξ are not observed but instead vectors $y' = (y_1, y_2, \ldots, y_p)$ and $x' = (x_1, x_2, \ldots, x_q)$ are observed, such that

$$y = \mu + \Lambda_y \eta + \epsilon \quad (2)$$

$$x = \nu + \Lambda_x \xi + \delta \quad (3)$$

where $\mu = \mathcal{E}(y)$, $\nu = \mathcal{E}(x)$ and ϵ and δ are vectors of errors of measurement in y and x, respectively. The matrices $\Lambda_y(p \times m)$ and $\Lambda_x(q \times n)$ are regression matrices of y on η and of x on ξ, respectively. It is convenient to refer to y and x as the observed variables and η and ξ as the true variables. The errors of measurement are assumed to be uncorrelated with each other and with the true variates.

Let $\Phi(n \times n)$ and $\Psi(m \times m)$ be the variance-covariance matrices of ξ and ζ, respectively, Θ_ϵ^2 and Θ_δ^2 the diagonal matrices of error variances for y and x, respectively. Then it follows, from the above assumptions, that the variance-covariance matrix $\Sigma[(p+q) \times (p+q)]$ of $z = (y', x')'$ is

$$\Sigma = \begin{pmatrix} \Lambda_y(B^{-1}\Gamma\Phi\Gamma'B'^{-1} + B^{-1}\Psi B'^{-1})\Lambda_y' + \Theta_\epsilon^2 & \Lambda_y B^{-1}\Gamma\Phi\Lambda_x' \\ \Lambda_x\Phi\Gamma'B'^{-1}\Lambda_y' & \Lambda_x\Phi\Lambda_x' + \Theta_\delta^2 \end{pmatrix} \qquad (4)$$

The elements of Σ are functions of the elements of Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_δ, and Θ_ϵ. In applications some of these elements are fixed and equal to assigned values. In particular this is so for elements in Λ_y, Λ_x, B and Γ, but we shall allow for fixed values in the other matrices also. For the remaining nonfixed elements of the six parameter matrices one or more subsets may have identical but unknown values. Thus elements in Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_δ, and Θ_ϵ are of three kinds: (i) fixed parameters that have been assigned given values, (ii) constrained parameters that are unknown but equal to one or more other parameters and (iii) free parameters that are unknown and not constrained to be equal to any other parameter.

Before an attempt is made to estimate a model of this kind, the identification problem must be examined. Identifiability depends on the specification of fixed, constrained, and free parameters. Under a given specification, a given structure Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_δ, Θ_ϵ generates one and only one Σ but there may be several structures generating the same Σ. If two or more structures generate the same Σ, the structures are said to be equivalent. If a parameter has the same value in all equivalent structures, the parameter is said to be identified. If all parameters of the model are identified, the whole model is said to be identified. When a model is identified one can usually find consistent estimates of all its parameters. Identification problems under some special cases of the general model are considered by Geraci and Goldberger (1971).

1.2. Estimation of the General Model

It is assumed that $\underset{\sim}{z} = (\underset{\sim}{y}', \underset{\sim}{x}')'$ has a multivariate normal distribution with mean vector $(\underset{\sim}{\mu}', \underset{\sim}{\nu}')'$ and variance-covariance matrix Σ.

Let $\underset{\sim}{z}_1, \underset{\sim}{z}_2, \ldots, \underset{\sim}{z}_M$ be M observations of $\underset{\sim}{z} = (\underset{\sim}{y}', \underset{\sim}{x}')'$. Since no constraints are imposed on the mean vector $(\underset{\sim}{\mu}', \underset{\sim}{\nu}')'$ the maximum likelihood estimate of this is the usual sample mean vector $\bar{\underset{\sim}{z}} = (\bar{\underset{\sim}{y}}', \bar{\underset{\sim}{x}}')'$. Let

$$\underset{\sim}{S} = \frac{1}{N} \sum_{\alpha=1}^{N} (\underset{\sim}{z}_\alpha - \bar{\underset{\sim}{z}})(\underset{\sim}{z}_\alpha - \bar{\underset{\sim}{z}})' \quad , \qquad (5)$$

with $N = M - 1$, be the usual sample variance-covariance matrix, partitioned as

$$\underset{\sim}{S}[(p+q) \times (p+q)] = \begin{bmatrix} \underset{\sim}{S}_{yy}(p \times p) & \underset{\sim}{S}_{yx}(p \times q) \\ \underset{\sim}{S}_{xy}(q \times p) & \underset{\sim}{S}_{xx}(q \times q) \end{bmatrix} \qquad (6)$$

The logarithm of the likelihood function, omitting a function of the observations, is given by

$$\log L = -\frac{1}{2} N[\log|\underset{\sim}{\Sigma}| + \mathrm{tr}(\underset{\sim}{S}\underset{\sim}{\Sigma}^{-1})] \qquad (7)$$

This is regarded as a function of the independent distinct parameters in Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_δ and Θ_ε and is to be maximized with respect to these, taking into account that some elements may be fixed and some may be constrained to be equal to some others. Maximizing $\log L$ is equivalent to minimizing

$$F = (1/2)[\log|\Sigma| + \mathrm{tr}(S\Sigma^{-1}) - \log|S| - (p + q)] \quad (8)$$

Such a minimization problem may be formalized as follows.

Let $\lambda' = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ be a vector of all the elements of Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_δ, and Θ_ε arranged in a prescribed order. Then F may be regarded as a function $F(\lambda)$ of $\lambda_1, \lambda_2, \ldots, \lambda_r$, which is continuous and has continuous derivatives $\partial F/\partial\lambda_s$ and $\partial^2 F/\partial\lambda_s\partial\lambda_t$ of first and second order, except where Σ is singular. The totality of these derivatives is represented by a gradient vector $\partial F/\partial\lambda$ and a symmetric matrix $\partial^2 F/\partial\lambda\partial\lambda'$. Now let some $r - s$ of the λ's be fixed and denote the remaining λ's by $\pi_1, \pi_2, \ldots, \pi_s$, $s \leq r$. The function F is now considered as a function $G(\pi)$ of $\pi_1, \pi_2, \ldots, \pi_s$. Derivatives $\partial G/\partial\pi$ and $\partial^2 G/\partial\pi\partial\pi'$ are obtained from $\partial F/\partial\lambda$ and $\partial^2 F/\partial\lambda\partial\lambda'$ by omitting rows and columns corresponding to the fixed λ's. Among $\pi_1, \pi_2, \ldots, \pi_s$, let there be some t distinct parameters denoted $\kappa_1, \kappa_2, \ldots, \kappa_t$, $t \leq s$, so that each π_i is equal to one and only one κ_j, but possibly several π's equal the same κ. Let $K = (k_{ij})$ be a matrix of order $s \times t$ with elements $k_{ij} = 1$ if $\pi_i = \kappa_j$ and $k_{ij} = 0$ otherwise. The function F (or G) is now a function $H(\kappa)$ of $\kappa_1, \kappa_2, \ldots, \kappa_t$ and we have

$$\partial H/\partial \kappa = K'(\partial G/\partial \pi) \qquad (9)$$

$$\partial^2 H/\partial \kappa \partial \kappa' = K'(\partial^2 G/\partial \pi \partial \pi')K \qquad (10)$$

Thus, the derivatives of H are simple sums of the derivatives of G.

The minimization of $H(\kappa)$ is now a straightforward application of the Davidon-Fletcher-Powell method (Fletcher & Powell, 1963) using a computer program by Gruvaeus and Jöreskog (1970). This method makes use of a matrix E, which is evaluated in each iteration. Initially E is any positive definite matrix approximating the inverse of $\partial^2 H/\partial \kappa \partial \kappa'$. In subsequent iterations E is improved, using the information built up about the function so that ultimately E converges to an approximation of the inverse of $\partial^2 H/\partial \kappa \partial \kappa'$ at the minimum. If there are many parameters, the number of iterations may be excessive, but can be considerably decreased by the provision of a good initial estimate of E. Such an estimate may be obtained by inverting the information matrix

$$\mathcal{E}(\partial^2 H/\partial \kappa \partial \kappa') = K'\mathcal{E}(\partial^2 G/\partial \pi \partial \pi')K \qquad (11)$$

where $\mathcal{E}(\partial^2 G/\partial \pi \partial \pi')$ is obtained from

$$\mathcal{E}(\partial^2 F/\partial \lambda \partial \lambda') = N\mathcal{E}(\partial F/\partial \lambda\, \partial F/\partial \lambda') \qquad (12)$$

by omitting rows and columns corresponding to the fixed λ's. When the minimum of H has been found, the inverse of the information matrix may be computed again to obtain standard errors of all the parameters in κ. A general method for obtaining the elements of $\mathcal{E}(\partial F/\partial \lambda \partial F/\partial \lambda')$ has been given by Jöreskog (1973).

In the program to be described here the information matrix is not used. To use the information matrix in the program would require the writing of a fairly complicated subroutine. This will possibly be done at a later time. At present the program works as follows. The starting point may be chosen arbitrarily. From the starting point a number of steepest descent iterations are performed until the decrease in function values is less than 5%. At the new point, so obtained, the Davidon-Fletcher-Powell procedure starts with $E^{(1)}$ chosen as an identity matrix.

The application of the Davidon-Fletcher-Powell method requires formulas for the derivatives of F with respect to the elements of Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_δ, and Θ_ϵ. These may be obtained by matrix differentiation. Writing $A = B^{-1}$, $D = B^{-1}\Gamma$, $C = D\Phi D' + A\Psi A'$, and

$$\Omega = \begin{pmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{pmatrix} = \Sigma^{-1}(\Sigma - S)\Sigma^{-1} , \qquad (13)$$

the derivatives are

$$\partial F/\partial \Lambda_y = \Omega_{yy}\Lambda_y C + \Omega'_{yx}\Lambda_x D'$$

$$\partial F/\partial \Lambda_x = \Omega_{xy}\Lambda_y D\Phi + \Omega_{xx}\Lambda_x \Phi$$

$$\partial F/\partial B = -A'\Lambda_y'(\Omega_{yy}\Lambda_y C + \Omega'_{xy}\Lambda_x \Phi D')$$

$$\partial F/\partial \Gamma = A'\Lambda_y'(\Omega_{yy}\Lambda_y D + \Omega'_{xy}\Lambda_x)\Phi$$

$$\partial F/\partial \Phi = Z - \mathrm{diag}(Z) ,$$

where

$$\underset{\sim}{Z} = \underset{\sim}{D}'\underset{\sim y}{\Lambda}'\underset{\sim yy}{\Omega}\underset{\sim y}{\Lambda}\underset{\sim}{D} + \underset{\sim x}{\Lambda}'\underset{\sim xy}{\Omega}\underset{\sim y}{\Lambda}\underset{\sim}{D} + \underset{\sim}{D}'\underset{\sim y}{\Lambda}'\underset{\sim yx}{\Omega}\underset{\sim x}{\Lambda} + \underset{\sim x}{\Lambda}'\underset{\sim xx}{\Omega}\underset{\sim x}{\Lambda}$$

$$\partial F/\partial \underset{\sim}{\Psi} = \underset{\sim}{A}'\underset{\sim y}{\Lambda}'\underset{\sim yy}{\Omega}\underset{\sim y}{\Lambda}\underset{\sim}{A} - \mathrm{diag}(\underset{\sim}{A}'\underset{\sim y}{\Lambda}'\underset{\sim yy}{\Omega}\underset{\sim y}{\Lambda}\underset{\sim}{A})$$

$$\partial F/\partial \underset{\sim}{\Theta}_{\epsilon} = \mathrm{diag}(\underset{\sim yy}{\Omega}\underset{\sim}{\Theta}_{\epsilon})$$

$$\partial F/\partial \underset{\sim}{\Theta}_{\delta} = \mathrm{diag}(\underset{\sim xx}{\Omega}\underset{\sim}{\Theta}_{\delta})$$

1.5. Tests of Hypotheses

When the maximum likelihood estimates of the parameters have been obtained, the goodness of fit of the model may be tested, in large samples, by the likelihood ratio technique. Let H_0 be the null hypothesis of the model under the given specifications of fixed, constrained, and free parameters. First consider the case when the alternative hypothesis H_1 is that $\underset{\sim}{\Sigma}$ is any positive definite matrix. Then minus twice the logarithm of the likelihood ratio is NF_0 where F_0 is the minimum value of F. If the model holds, this is distributed, in large samples, as χ^2 with

$$d = \frac{1}{2}(m + n)(m + n + 1) - t \tag{1.-}$$

degrees of freedom, where, as before, t is the total number of independent parameters estimated under H_0.

Let H_0 be any specific hypothesis concerning the parametric structure of the general model and let H_1 be an alternative hypothesis. In large samples one can then test H_0 against H_1. Let F_0 be the minimum of F under H_0 and let F_1 be the minimum of F under H_1. Then $F_1 \leq F_0$ and minus twice the logarithm of the likelihood ratio becomes $N(F_0 - F_1)$. Under H_0 this is distributed approximately as χ^2 with degrees of freedom equal to the difference in number of independent parameters estimated under H_1 and H_0.

4. A Hypothetical Model

To illustrate the ideas of the preceding sections consider the model depicted in Figure 1, where circles denote true variables and squares denote observed variables. The other variables in the figure are residuals or error variables. A one-way arrow denotes a direct causal influence whereas a double arrow denotes correlation or covariation without a causal interpretation. The two arrows between η_1 and η_2 denote reciprocal interaction (simultaneity or interdependence).

The model in Figure 1 has $p = 4$ y-variables, $q = 7$ x-variables, $m = 2$ η-variables, and $n = 3$ ξ-variables. The structural equations are

$$\eta_1 = -\beta_1\eta_2 + \gamma_1\xi_1 + \gamma_1\xi_2 + \zeta_1 \quad ,$$

$$\eta_2 = -\beta_2\eta_1 - \xi_1 + \gamma_2\xi_3 + \zeta_2 \quad ,$$

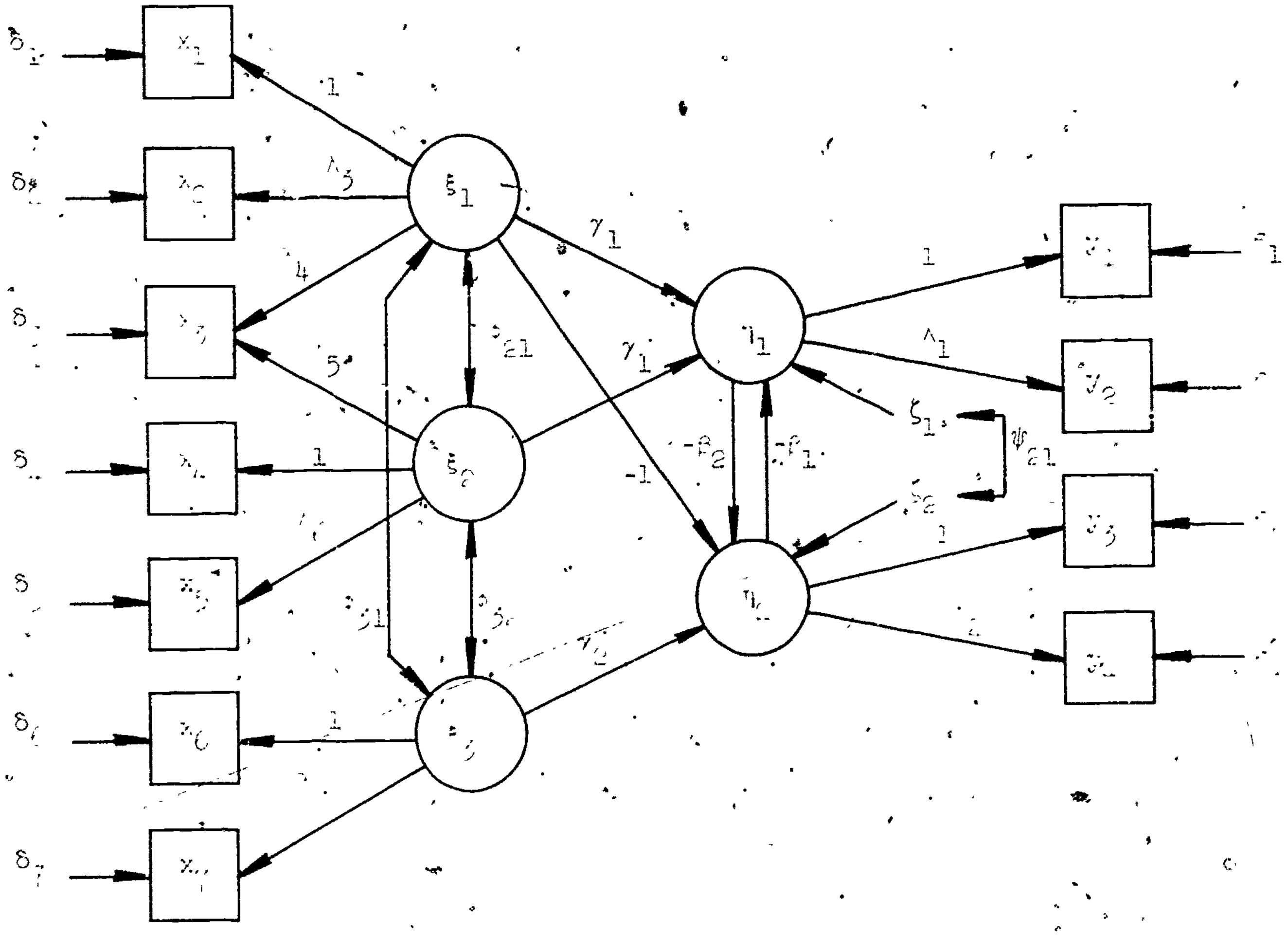
or

$$\begin{pmatrix} 1 & \beta_1 \\ \beta_2 & 1 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \gamma_1 & \gamma_1 & 0 \\ -1 & 0 & \gamma_2 \end{pmatrix}\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} \tag{1a}$$

The two γ-coefficients in the first equation are assumed to be equal to illustrate the idea of a constrained parameter. The equations relating the observed and true variables are

Figure 1

A Hypothetical Model



$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ \lambda_1 & 0 \\ 0 & 1 \\ 0 & \lambda_2 \end{bmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{pmatrix} \tag{15b}$$

and

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \\ \nu_7 \end{pmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ \lambda_3 & 0 & 0 \\ \lambda_4 & \lambda_5 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_6 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \lambda_7 \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \end{pmatrix} \tag{15c}$$

In (15b) and (15c) one λ in each column of $\underset{\sim}{\Lambda}_y$ and $\underset{\sim}{\Lambda}_x$ has been set equal to one to fix the scales of measurement in the true variables. When a solution has been obtained, one can scale this so that all true variables have unit variance if this is desired.

Data for this model were generated by assigning the following values

$$\underset{\sim}{\Lambda}_y = \begin{bmatrix} 1.000 & 0.0 \\ 0.902 & 0.0 \\ 0.0 & 1.000 \\ 0.0 & 1.095 \end{bmatrix},$$

$$\underset{\sim}{\Lambda}_x = \begin{bmatrix} 1.000 & 0.0 & 0.0 \\ 1.300 & 0.0 & 0.0 \\ 0.900 & 1.201 & 0.0 \\ 0.0 & 1.000 & 0.0 \\ 0.0 & 1.098 & 0.0 \\ 0.0 & 0.0 & 1.000 \\ 0.0 & 0.0 & 1.400 \end{bmatrix},$$

$$\underset{\sim}{B} = \begin{bmatrix} 1.000 & -0.493 \\ -0.595 & 1.000 \end{bmatrix},$$

$$\underset{\sim}{\Gamma} = \begin{pmatrix} 0.399 & 0.399 & 0.0 \\ -1.000 & 0.0 & 1.198 \end{pmatrix} ,$$

$$\underset{\sim}{\Phi} = \begin{pmatrix} 0.999 & & \\ 0.700 & 1.199 & \\ 0.601 & 0.300 & 1.398 \end{pmatrix} ,$$

$$\underset{\sim}{\Psi} = \begin{pmatrix} 0.506 & \\ 0.386 & 0.705 \end{pmatrix} ,$$

$$\underset{\sim}{\Theta}_\epsilon = \text{diag}(0.522,\ 0.432,\ 0.356,\ 0.452) ,$$

$$\underset{\sim}{\Theta}_\delta = \text{diag}(0.613,\ 0.515,\ 0.418,\ 0.522,\ 0.614,\ 0.526,\ 0.414) ,$$

to each of the parameter matrices. These generate a $\underset{\sim}{\Sigma}$ according to (4), where

$$\underset{\sim}{\Sigma}_{yy} = \begin{bmatrix} 3.815 & & & \\ 3.195 & 3.068 & & \\ 3.890 & 3.508 & 5.773 & \\ 4.259 & 3.842 & 6.182 & 6.973 \end{bmatrix} ,$$

$$\underset{\sim}{\Sigma}_{xy} = \begin{bmatrix} 0.764 & 0.690 & 0.176 & 0.193 \\ 0.994 & 0.896 & 0.229 & 0.251 \\ 1.690 & 1.525 & 0.347 & 0.380 \\ 0.835 & 0.753 & 0.157 & 0.172 \\ 0.917 & 0.827 & 0.173 & 0.189 \\ 1.259 & 1.136 & 1.824 & 1.997 \\ 1.\text{[illegible]}63 & 1.591 & 2.554 & 2.797 \end{bmatrix} ,$$

$$\underset{\sim}{\Sigma}_{xx} = \begin{bmatrix} 1.374 & & & & & & \\ 1.298 & 1.952 & & & & & \\ 1.739 & 2.260 & 4.224 & & & & \\ 0.700 & 0.909 & 2.069 & 1.471 & & & \\ 0.768 & 0.998 & 2.272 & 1.317 & 1.822 & & \\ 0.601 & 0.781 & 0.902 & 0.300 & 0.330 & 1.675 & \\ 0.841 & 1.094 & 1.262 & 0.421 & 0.462 & 1.958 & 2.914 \end{bmatrix} .$$

In the Appendix this model is analyzed using the above matrices as $\underset{\sim}{S}_{yy}$, $\underset{\sim}{S}_{xy}$, and $\underset{\sim}{S}_{xx}$.

1.5. A Model of Duncan, Haller and Portes

In a study on peer influences on aspirations, Duncan, Haller, and Portes (1968) gave several examples of path analysis models. Their model IV is particularly interesting since it involves two unmeasured variables. This model is reproduced here, in different notation, in Figure 2. In this model, $p = 4$, $q = 6$, $m = 2$, $n = 6$. The six x-variables are assumed to be measured without error, so we take $\xi_i \equiv x_i - \nu_i$, $i = 1,2,\ldots,6$, i.e., in terms of equation (3) we have $\underset{\sim}{\Lambda}_x = \underset{\sim}{I}$ and $\underset{\sim}{\delta} = \underset{\sim}{0}$. The structural equations are

$$\begin{pmatrix} 1 & \beta_1 \\ \beta_2 & 1 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & 0 & 0 \\ 0 & 0 & \gamma_5 & \gamma_6 & \gamma_7 & \gamma_8 \end{pmatrix}\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix}, \qquad (16a)$$

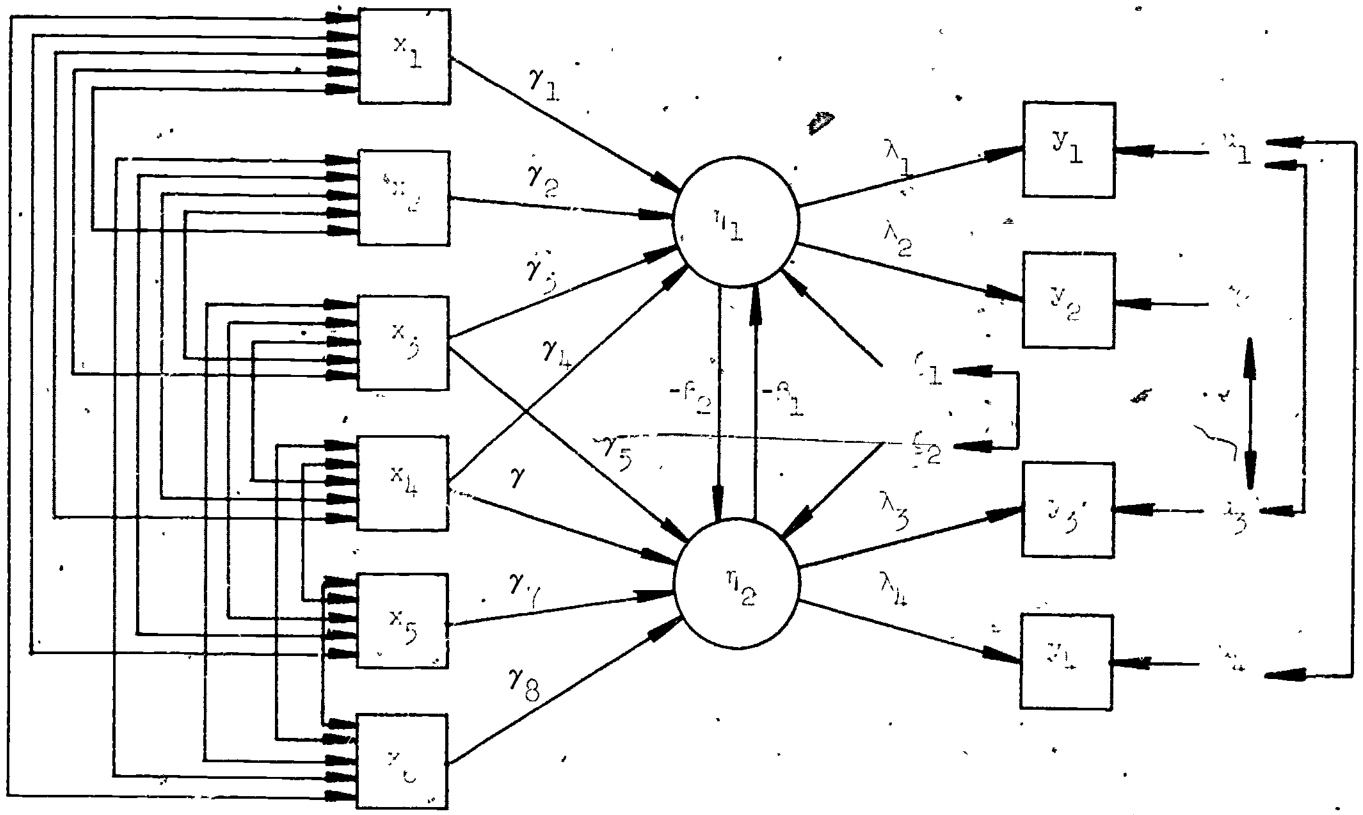
and the equations relating the η's to the y's are

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} + \begin{pmatrix} \lambda_1 & 0 \\ \lambda_2 & 0 \\ 0 & \lambda_3 \\ 0 & \lambda_4 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}. \qquad (16b)$$

Duncan, Haller, and Portes postulate that the u's are correlated with each other except that u_1 and u_2 are uncorrelated and also u_3 and u_4. However, the four correlations $\rho(u_1, u_3)$, $\rho(u_1, u_4)$, $\rho(u_2, u_3)$ and $\rho(u_2, u_4)$ are not all identified. To make the model identified one of them must be fixed and we have chosen to set $\rho(u_2, u_4) = 0$. Equation (16b) is not in the form required by the general model. This is easily remedied by representing $\underset{\sim}{u}$ as

Figure 2

A Model of Duncan, Haller, and Portes (1968)



$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} \lambda_5 & 0 & 0 & \lambda_6 \\ 0 & \lambda_7 & \lambda_8 & 0 \\ \lambda_9 & 0 & \lambda_{10} & 0 \\ 0 & 0 & 0 & \lambda_{11} \end{pmatrix} \begin{pmatrix} \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix} , \tag{16c}$$

where η_3, η_4, η_5, and η_6 are mutually uncorrelated and of unit variances. It should be noted that there is a one-to-one correspondence between the nonzero variances and covariances of $\underset{\sim}{u}$ and $\lambda_5, \lambda_6, \ldots, \lambda_{11}$. Introducing $\xi_7 \equiv \eta_3$, $\xi_8 \equiv \eta_4$, $\xi_9 \equiv \eta_5$, and $\xi_{10} \equiv \eta_6$ the whole model may be specified as follows

$$\begin{pmatrix} 1 & \beta_1 & 0 & 0 & 0 & 0 \\ \beta_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_5 & \gamma_6 & \gamma_7 & \gamma_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \\ \xi_9 \\ \xi_{10} \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} , \tag{17a}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \\ \xi_9 \\ \xi_{10} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} , \tag{17b}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} + \begin{bmatrix} \lambda_1 & 0 & \lambda_5 & 0 & 0 & \lambda_6 \\ \lambda_2 & 0 & 0 & \lambda_7 & \lambda_8 & 0 \\ 0 & \lambda_3 & \lambda_9 & 0 & \lambda_{10} & 0 \\ 0 & \lambda_4 & 0 & 0 & 0 & \lambda_{11} \end{bmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (17c)$$

Furthermore one specifies

$$\underset{\sim}{\Phi}(10 \times 10) = \begin{pmatrix} \underset{\sim}{\Sigma}_{xx}(6 \times 6) & \\ \underset{\sim}{0}(4 \times 6) & \underset{\sim}{I}(4 \times 4) \end{pmatrix},$$

$$\underset{\sim}{\Psi}(6 \times 6) = \begin{bmatrix} 1 & & & & & \\ \rho(\zeta_1\zeta_2) & 1 & & & & \\ 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\underset{\sim}{\Theta}_\delta(6 \times 6) = \underset{\sim}{0}$$

and

$$\underset{\sim}{\Theta}_\epsilon(4 \times 4) = \underset{\sim}{0} \quad .$$

This model is analyzed in the appendix using the following data from Table 1 of Duncan, Haller, and Portes (1968) (N = 329):

$\underset{\sim}{S}$ =

x_2	x_1	x_3	y_1	y_2	x_5	x_6	x_4	y_4	y_3
1.0000	.1839	.2220	.4105	.4043	.3355	.1021	.1861	.2598	.2903
	1.0000	.0489	.2137	.2742	.0782	.1147	.0186	.0839	.1124
		1.0000	.3240	.4047	.2302	.0931	.2707	.2786	.3054
			1.0000	.6247	.2995	.0760	.2930	.4216	.3269
				1.0000	.2863	.0702	.2407	.3275	.3669
					1.0000	.2087	.2950	.5007	.5191
						1.0000	-.0438	.1988	.2784
							1.0000	.3607	.4105
								1.0000	.6404
									1.0000

1.6. A Restricted Factor Analysis Model

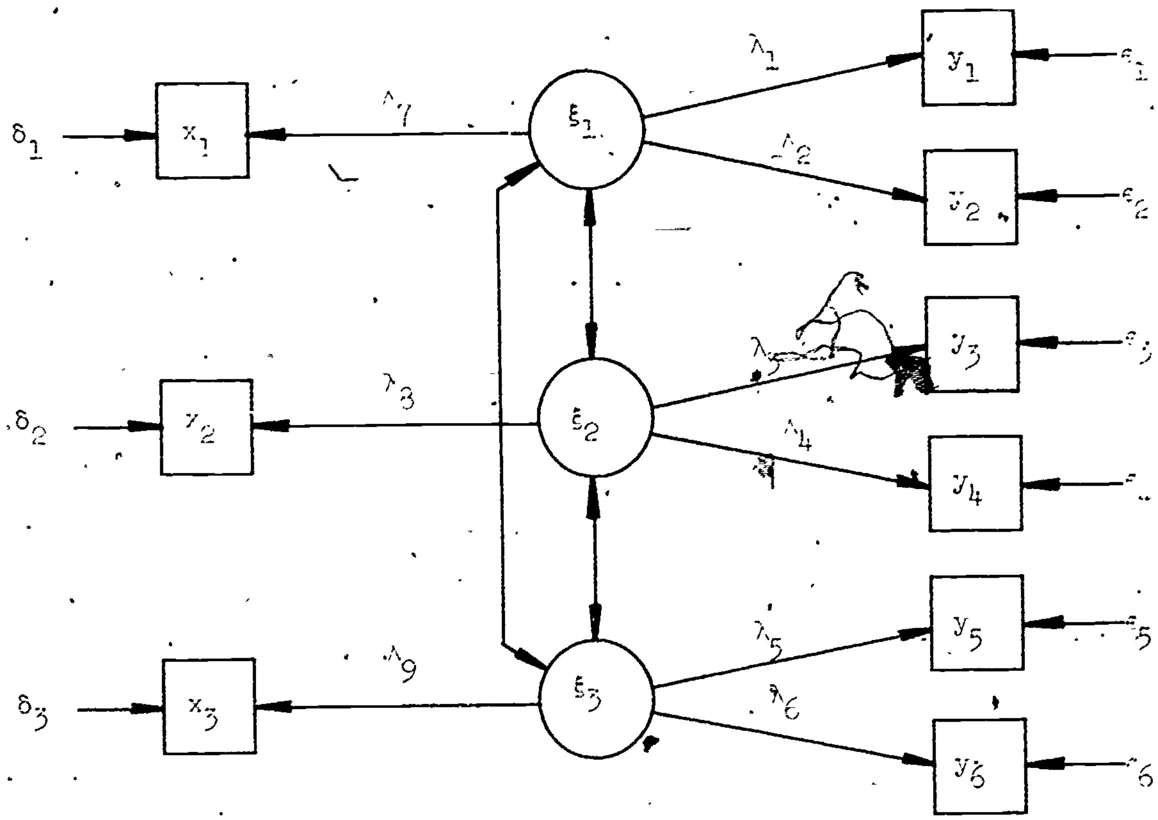
Consider the model shown in Figure 3. This is a restricted factor analysis model in the sense of Jöreskog (1969) and may be most conveniently analyzed by the method described in that paper. However, it can also be analyzed using the model considered in this paper. To do so one specifies $\underset{\sim}{\eta} \equiv \underset{\sim}{\xi}$, i.e., one chooses $\underset{\sim}{B}(3 \times 3) = \underset{\sim}{I}$, $\underset{\sim}{\Gamma}(3 \times 3) = \underset{\sim}{I}$, and $\underset{\sim}{\zeta} = \underset{\sim}{0}$. The matrices $\underset{\sim}{\Lambda}_y$ and $\underset{\sim}{\Lambda}_x$ are as follows:

$$\underset{\sim}{\Lambda}_y = \begin{bmatrix} \lambda_1 & 0 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & \lambda_4 & 0 \\ 0 & 0 & \lambda_5 \\ 0 & 0 & \lambda_6 \end{bmatrix}, \quad \underset{\sim}{\Lambda}_x = \begin{bmatrix} \lambda_7 & 0 & 0 \\ 0 & \lambda_8 & 0 \\ 0 & 0 & \lambda_9 \end{bmatrix}.$$

The matrix $\underset{\sim}{\Phi}$ is specified to be a correlation matrix and $\underset{\sim}{\Theta}_\epsilon^2$ and $\underset{\sim}{\Theta}_\delta^2$ contain the unique variances of the tests. An example is given in the Appendix.

Figure 3

A Restricted Factor Analysis Model



2. The Program

In this section we describe briefly what the program does. Details about the input and output are given in sections 3 and 4 respectively.

2.1. What the Program Does

The input data may be a dispersion matrix, a correlation matrix, a correlation matrix with standard deviations, or raw data from which the matrix to be analyzed can be computed. From the input matrix, variables may be selected to be excluded in the analysis, so that the matrix to be analyzed may be of smaller order than the input matrix. This selection procedure also allows columns (rows) of the input matrix to be interchanged.

The matrix to be analyzed may be sums of squares and cross products, deviation sums of squares and cross products, a dispersion matrix, or a correlation matrix.

Simplified versions of the general model may be estimated. That is, the user can specify to estimate a model where there are no disturbances, or a model where there is no $\underset{\sim}{x}$, or a model where $\underset{\sim}{B} = \underset{\sim}{I}$. In these cases the equations are simplified and the minimization procedure is faster. Also, the user can request an accurate or an approximate solution. If an accurate solution is requested, the iterations of the minimization method are continued until the minimum is found, the convergence criterion being that the magnitude of all derivatives be less than .00005. The solution is then usually correct to three significant digits. If an approximate solution is requested, the iterations terminate when the decrease in function values is less than 5%. The approximate solution may be substantially different from the exact solution, but the residuals and the value of χ^2 will usually give an indication of how reasonable the hypothesized model is. The option of an approximate solution has been included in the program for the purpose of saving computer time in exploratory studies when the primary purpose is to find a reasonable model. Once such a model has been found, an accurate solution may be computed.

A variety of options for the output is available. Residuals may be printed. These are defined as the difference between reproduced (Σ) and observed (S) variances and covariances, which are useful for judging the goodness of fit of the model to the data. The standardized solution can be computed and printed if requested. χ^2 is printed as an overall goodness of fit test statistic. The final maximum likelihood solution may be punched on cards if requested.

2.2. How Fixed, Free and Constrained Parameters Are Specified

The elements of the eight parameter matrices are assumed to be in the order Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_ϵ, Θ_δ and within each matrix, the elements are ordered row-wise. The diagonal elements of the diagonal matrices Θ_ϵ and Θ_δ are treated as row vectors and only the lower diagonal parts of symmetric Φ and Ψ are taken into account.

For each of the eight parameter matrices, a pattern matrix is defined, with elements 0, 1, 2, and 3 depending on whether the corresponding element in the parameter matrix is fixed, free, constrained follower and constrained leader, respectively. A constrained parameter is called a constrained leader the first time it appears in the sequence. The parameters, appearing later in the sequence and assumed to be equal to the constrained leader are called constrained followers.

The above technique defines uniquely the positions of the fixed, free and constrained leader parameters. It does not define, however, which followers go with which leader, if there is more than one leader. To do so one must also specify all the followers associated with a given leader. This is done by assigning to each leader and follower a four-digit number MCCC, where M defines the matrix in which the constrained parameter appears and CCC defines the position of the parameter in that matrix. Thus, $M = 1$ for $\underset{\sim}{\Lambda}_y$, 2 for $\underset{\sim}{\Lambda}_x$, 3 for $\underset{\sim}{B}$, 4 for $\underset{\sim}{\Gamma}$, 5 for $\underset{\sim}{\Phi}$, 6 for $\underset{\sim}{\Psi}$, 7 for $\underset{\sim}{\Theta}_\epsilon$, and 8 for $\underset{\sim}{\Theta}_\delta$. For example,

4001 4005 5014

defines the first element in $\underset{\sim}{\Gamma}$, γ_1, to be equal to the fifth element in $\underset{\sim}{\Gamma}$, γ_5, as well as the fourteenth element in $\underset{\sim}{\Phi}$, ϕ_{14}, where γ_1 is the leader and γ_5 and ϕ_{14} are the followers.

Pattern matrices have to be provided for each matrix containing both fixed and free parameters and for each matrix containing constrained parameters. Patterns for matrices whose elements are all fixed or all free are set up by the program.

We give a simple example to illustrate the above specifications. Suppose $\underset{\sim}{B}$ (3 x 3) = $\underset{\sim}{I}$ and $\underset{\sim}{\Gamma}$ (3 x 3) = $\underset{\sim}{I}$, all elements in both $\underset{\sim}{B}$ and $\underset{\sim}{\Gamma}$ fixed, $\underset{\sim}{\Theta}_\delta$ (3 x 3) with all diagonal elements free, and

$$\underset{\sim}{\Lambda}_y = \begin{bmatrix} \lambda_1 & 0 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & \lambda_4 & 0 \\ 0 & 0 & \lambda_5 \\ 0 & 0 & \lambda_6 \end{bmatrix} \quad \underset{\sim}{\Lambda}_x = \begin{bmatrix} \lambda_7 & 0 & 0 \\ 0 & \lambda_8 & 0 \\ 0 & 0 & \lambda_9 \end{bmatrix} \quad \underset{\sim}{\Phi} = \begin{bmatrix} 1 & & \\ \phi_1 & 1 & \\ \phi_2 & \phi_3 & 1 \end{bmatrix}$$

$$\text{diag}\ (\underset{\sim}{\Theta}_\epsilon) = (\theta_{\epsilon_1}, \theta_{\epsilon_2}, \theta_{\epsilon_3}, \theta_{\epsilon_4}, \theta_{\epsilon_5}, \theta_{\epsilon_6})$$

with $\lambda_1 = \lambda_2 = \lambda_8 = \phi_2$, $\lambda_5 = \lambda_6$ and $\theta_{\epsilon_1} = \theta_{\epsilon_2}$, $\theta_{\epsilon_3} = \theta_{\epsilon_4}$, $\theta_{\epsilon_5} = \theta_{\epsilon_6}$. The pattern matrices for Λ_y , Λ_x , Φ and Θ_δ are

$$P_{\Lambda_y} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix} \quad P_{\Lambda_x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad P_{\Phi} = \begin{bmatrix} 0 & & \\ 1 & 0 & \\ 2 & 1 & 0 \end{bmatrix}$$

$$P_{\Theta_\epsilon} = [\,3\ 2\ 3\ 2\ 3\ 2\,]$$

and the specifications of leaders and followers are

1001	1004	2005	5004
1015	1018		
7001	7002		
7003	7004		
7005	7006		

In this model fourteen independent parameters will be estimated.

In addition to the above specifications for fixed, free and constrained parameters, start values have to be given for all parameter matrices, except when a simplified model is to be estimated (see 2.1). That is, if there are no disturbances, start values for Ψ are not read in. If $B = I$, B is not read in. And if there is no x start values for Λ_x , Γ , Φ and Θ_δ are not read in. The start values define the fixed parameters and the initial values for the minimization procedure for the other parameters. Constrained parameters assumed to be equal must be given the same values. Otherwise, start values may be chosen arbitrarily but the closer they are to the final solution the less computer time it will take to reach the solution.

2.3. Limitations

The program is written in FORTRAN IV - G and has been tested out on the IBM 360/65 at Educational Testing Service. Double precision is used in floating point arithmetic throughout the program. With minor changes the program should run on any computer with a FORTRAN IV compiler. In computers with a single word length of 36 bits or more, single precision is probably sufficient.

Limitations as to the maximum number of variables, the maximum number of independent and nonfixed parameters, and the maximum order of the parameter matrices allowed, as well as the core requirements of the program on the IBM 360/65 are given in the following table.

max. no. of variables $(p_0 + q_0)$ before selection	= 80
max. no. of variables $(p + q)$ after selection	= 30
max. p	= 15
max. q	= 15
max. m	= 15
max. n	= 15
max. no. of independent parameters	= 80
max. no. of nonfixed parameters	= 80
core requirements (K = 1024 bytes = 256 words)	= 140K

2.4. Availability

A copy of the program may be obtained by writing to Marielle van Thillo at ETS. The user must provide a tape on which the program will be loaded. The program will be written on the tape with 80 characters per record. The tape will be unlabeled. The user must specify whether he wants the tape blocked or unblocked, on 7-track or 9-track, in EBCDIC or BCD mode, as well as the density and parity required. Test data will be at the end of the program. The test data are described in the Appendix. Anyone using the program for the first time should make sure that the test data run correctly.

2.5. Disclaimer

Although the program has been working satisfactorily for all data analyzed so far, no claim is made that it is free of error and no warranty is given as to the accuracy and functioning of the program.

3. Input Data

For each data to be analyzed, the input consists of the following:

1. Title card
2. Parameter card
3. Input matrix
4. Specifications for selection of variables from the input matrix
5. Pattern matrices for the parameter matrices
6. Equalities
7. Start values for the parameter matrices
8. New data set or a STOP card

Sections 3.1 through 3.8 describe in general terms the function and setup of each of the above quantities. Illustrative examples are given in the Appendix.

Whenever a matrix or vector is read in it is preceded by a format card, containing at most 80 columns, beginning with a left parenthesis and ending with a right parenthesis. The format must specify floating point numbers for the input matrix and parameter matrices, and fixed point numbers for the pattern matrices, consistent with the way in which the elements of the matrix are punched on the following cards. Users who are unfamiliar with FORTRAN are referred to a FORTRAN Manual, where format rules are given. Matrices are to be punched as one long vector, reading row-wise. For the symmetric matrices only the lower half of the matrix including the diagonal should be punched.

3.1. Title Card

Whatever appears on this card will appear on the first page of the printed output. All 80 columns of the card are available to the user.

5.2. Parameter Card

All quantities on this card, except for the logical indicators, must be punched as integers right adjusted within the field.

cols. 1 - 5	number of variables (p_0) in $\underset{\sim}{y}$ before selection
cols. 6 - 10	number of variables (q_0) in $\underset{\sim}{x}$ before selection (i.e., the input matrix $\underset{\sim}{S}$ is of order $(p_0 + q_0) \times (p_0 + q_0)$ before selection of variables)
cols. 11 - 15	number of columns in $\underset{\sim}{\Lambda}_y$ (m)
cols. 16 - 20	number of columns in $\underset{\sim}{\Lambda}_x$ (n)
cols. 21 - 25	number of observations (M)
cols. 26 - 30	total estimated execution time in seconds for all stacked data (SEC). This should be a number slightly less than the time requested on the control cards so that the program will have time to print and/or punch results up to that point (Note: SEC should be read in for each data set and should be the same for all data sets in the stack.)
col. 41	logical variable = F if there is no $\underset{\sim}{x}$, i.e., there are no $\underset{\sim}{\Lambda}_x$, $\underset{\sim}{\Gamma}$, $\underset{\sim}{\Phi}$, $\underset{\sim}{\Theta}_\delta$. (In this case n = q = 0). = T otherwise
col. 42	logical variable = F if there are no disturbances $\underset{\sim}{\zeta}$, i.e., there is no $\underset{\sim}{\psi}$ = T otherwise

col. 43	logical variable
	= F if $\underset{\sim}{B} = \underset{\sim}{I}$
	= T otherwise
col. 44	logical variable
	= F if the <u>exact</u> solution is to be computed
	= T if the <u>approximate</u> solution is to be computed
col. 45	logical variable
	= F if the solution is <u>not</u> to be punched on cards
	= T if the solution is to be punched on cards. This will automatically be done if IND $\neq$ 0 (see 4.1 and 4.6)
col. 46	logical variable
	= F if variables are <u>not</u> to be selected from the input matrix
	= T if variables are to be selected from the input matrix
col. 47	logical variable
	= F if the standardized solution is <u>not</u> to be printed
	= T if the standardized solution is to be printed (see 4.4)
col. 51	= 1 if raw data ($\underset{\sim}{Y} \mid \underset{\sim}{X}$) is read in to compute the matrix $\underset{\sim}{S}$ to be analyzed
	= 2 if the input matrix is a dispersion matrix
	= 3 if the input matrix is a correlation matrix
	= 4 if the matrix to be analyzed is the same as in the previous data set, i.e., the input matrix is <u>not</u> to be read in

col. 52 = 1 if the matrix to be analyzed is sums of squares and cross products

= 2 if the matrix to be analyzed is deviation sums of squares and cross products

<u>Note</u>: Col. 52 can be 1 or 2 <u>only</u> if col. 51 = 1

= 3 if the matrix to be analyzed is a dispersion matrix

= 4 if the matrix to be analyzed is a correlation matrix

col. 53 = 0 if only the standard output is to be printed (see 4.1)

= 1 if the input matrix, the specification matrices and the initial solution are to be printed

= 2 if $\underset{\sim}{\Sigma}$, the residuals and matrices $\underset{\sim}{C}$ and $\underset{\sim}{D}$ are to be printed (see 4.3)

= 3 if both 1 and 2 apply

= 4 if technical output from the minimization procedure is to be printed (see 4.5)

= 5 if both 1 and 4 apply

= 6 if both 2 and 4 apply

= 7 if 1, 2 and 4 apply

3.3. Input Matrix

If <u>col. 51 = 1</u> on the parameter card an $M \times (p_0 + q_0)$ matrix $(\underset{\sim}{Y}|\underset{\sim}{X})$ of raw data is read in, one row at a time, <u>starting a new card for each row.</u> Note that this is the only input matrix not read in as one continuous long vector. The matrix is preceded by a format card.

If col. 51 = 2 or 3 on the parameter card the lower triangular part, including the diagonal, of the input matrix $\underset{\sim}{S}$ is read in, reading row-wise. By $\underset{\sim}{S}$ we mean the partitioned matrix

$$\underset{\sim}{S}(p_0 + q_0 \times p_0 + q_0) = \begin{pmatrix} \underset{\sim}{S}_{yy}(p_0 \times p_0) & \\ \underset{\sim}{S}_{xy}(q_0 \times p_0) & \underset{\sim}{S}_{xx}(q_0 \times q_0) \end{pmatrix}$$

The input matrix is preceded by a format card. If a correlation matrix is read in but a dispersion matrix is to be analyzed (i.e., col. 51 = 3 and col. 52 = 3), the input matrix $\underset{\sim}{S}$ is followed by a format card and a vector of standard deviations on subsequent cards.

If col. 51 = 4 on the parameter card the input matrix is not to be read in.

3.4. Specifications for Selection of Variables from $\underset{\sim}{S}$

These cards will be read in only if column 46 = T and column 51 $\neq$ 4 on the parameter card. Omit otherwise.

The first card will have the integer values p and q punched in columns 1-5 and 6-10 respectively, right adjusted within the field. These integers will specify the order of $\underset{\sim}{S}$ after selection ($p + q \leq p_0 + q_0$).

The next card(s) will contain integers, right adjusted in five column fields (i.e., sixteen such values will fit on one card) specifying which columns (rows) are to be included. For example, if $p_0 = 6$, $q_0 = 3$ and $p = 3$, $q = 1$ and columns (rows) 1, 2, 5, 8 and 9 of S are to be excluded, then this card will have a 3 punched in column 5, and a 4 punched in column 10, a 6 punched in column 15 and a 7 punched in column 20.

Note: If $p_0 + q_0 = p + q$ there will be no reduction in the size of $\underset{\sim}{S}$ but columns (rows) can be interchanged.

3.5. Pattern Matrices for the Parameter Matrices

The pattern matrices are preceded by a data card with entries in columns 1-8, the column defining the matrix in question, 1 for $\underset{\sim}{\Lambda}_y$, 2 for $\underset{\sim}{\Lambda}_x$, 3 for $\underset{\sim}{B}$, 4 for $\underset{\sim}{\Gamma}$, 5 for $\underset{\sim}{\Phi}$, 6 for $\underset{\sim}{\Psi}$, 7 for $\underset{\sim}{\Theta}_\epsilon$, 8 for $\underset{\sim}{\Theta}_\delta$.

cols. 1 - 8 CCCCCCCC where C = 0 if the matrix is fixed
C = 1 if the matrix is free
C = 2 if the matrix has mixed values and/or constraints

A pattern matrix should be provided only when C = 2. (See 2.2.)

For example, if columns 1 - 8 are punched 22002021 the matrices $\underset{\sim}{\Lambda}_y$, $\underset{\sim}{\Lambda}_x$, $\underset{\sim}{\Phi}$, $\underset{\sim}{\Theta}_\epsilon$ contain mixed values and/or constraints, the matrices $\underset{\sim}{B}$, $\underset{\sim}{\Gamma}$, $\underset{\sim}{\Psi}$ are all fixed and matrix $\underset{\sim}{\Theta}_\delta$ is all free. In this case only pattern matrices for $\underset{\sim}{\Lambda}_y$, $\underset{\sim}{\Lambda}_x$, $\underset{\sim}{\Phi}$ and $\underset{\sim}{\Theta}_\epsilon$ are to be read in.

The pattern matrix consists of a format card specifying an I-format and subsequent cards with the integer entries of the pattern matrix.

3.6. Equalities

Omit if the pattern matrices do not contain any elements 2 or 3. Otherwise starting in column 1 punch the four-digit numbers MCCC as described in section 2.2. For each new constrained leader start a new card. The last entry on each "equality" card should be a zero indicating more "equality" cards are to follow or a nine indicating it is the last one. In the example used in section 2.2 these cards would look as follows:

100110040
100810110
101510180
700170020
700370040
700570069

3.7. Start Values for the Parameter Matrices

Start values for each of the parameter matrices are read in, each preceded by a format card, and in the order previously described (i.e., Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_ϵ, Θ_δ). Only the lower half of Φ and Ψ are read in.

If col. 41 = F on the parameter card do not read in start values for Λ_x, Γ, Φ and Θ_δ.

If col. 42 = F on the parameter card do not read in a start value for Ψ.

If col. 43 = F on the parameter card do not read in a start value for B.

3.8. Stacked Data

In sections 3.1 to 3.7 we have described how each set of data should be punched. Any number of such sets of data may be stacked together and analyzed in one run. After the last set of data in the stack, there must be a card with the word STOP punched in columns 1 - 4.

4. Printed and Punched Output

The output consists of a series of printed and punched tables as described in sections 4.1 - 4.5. Examples of printed outputs are given in the Appendix.

4.1. Standard Output

The standard output is always obtained, regardless of the value punched in column 53 of the parameter card (see 3.2). The standard output consists of the title with parameter listing, the final solution and the result of the test of goodness of fit.

The parameter listing gives the information supplied on the parameter card.

The final solution consists of the eight matrices Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_ϵ, Θ_δ. All numbers are printed with three decimals.

The test of goodness of fit gives the value of χ^2 and the corresponding degrees of freedom. The probability level is also given. This is defined as the probability of getting a χ^2 value larger than that actually obtained, given that the hypothesized model is true.

Just above the table giving the final solution, the following message is printed

"IND = X"

Usually X = 0, but if, for some reason, it has not been possible to determine the final solution, X will be 1, 2, 3, 4 or 5. If IND is 1, 2 or 3, "serious problems" have been encountered and the minimization the function cannot continue. One reason for this may be erroneous input data. Another reason may be that insufficient arithmetic precision is used. If IND is 4, the number of iterations has exceeded 250. If IND is 5, the time limit SEC has been exceeded (see 3.2). If IND $\neq 0$, the solution obtained so far is automatically punched on cards in such a way as to be immediately available as initial estimates for a new run with the same data. Thus there is little loss of information when execution is terminated with IND $\neq 0$.

4.2. Input Matrix $\underset{\sim}{S}$ and Parameter Specifications

If column 53 of the parameter card is 1, 3, 5 or 7 (see 3.2), the matrix to be analyzed, $\underset{\sim}{S}$, as obtained after exclusion or interchanging of variables, if any, is printed. By $\underset{\sim}{S}$ we mean the partitioned matrix

$$\underset{\sim}{S} = \begin{pmatrix} \underset{\sim}{S}_{yy} & \\ \underset{\sim}{S}_{xy} & \underset{\sim}{S}_{xx} \end{pmatrix} .$$

The matrix $\underset{\sim}{S}_{yy}$ will be printed first, followed by $\underset{\sim}{S}_{xy}$ and $\underset{\sim}{S}_{xx}$.

A table of parameter specifications, containing the information provided by the pattern matrices and equality cards (see 2.2) is also printed. Integer matrices are printed corresponding to the parameter matrices. In each matrix an element is an integer equal to the index of the corresponding parameter in the sequence of independent parameters. Elements corresponding to fixed parameters are 0 and elements corresponding to the same constrained parameter have the same value. Examples are given in the Appendix.

The initial solution or start values for the parameter matrices will also be printed.

4.3. Matrices $\hat{\Sigma}$, $\hat{C}$, $\hat{D}$ and Residuals

If column 53 of the parameter card is 2, 3, 6 or 7 (see 3.2), the matrices $\hat{\Sigma}$, $\hat{C}$, $\hat{D}$ and residuals are printed.

The matrices $\hat{\Sigma}$, $\hat{C}$, $\hat{D}$ are computed from the final solution. By $\hat{\Sigma}$, $\hat{C}$, $\hat{D}$ we mean

$$\hat{\Sigma} = \begin{pmatrix} \hat{\Sigma}_{yy} & \\ \hat{\Sigma}_{xy} & \hat{\Sigma}_{xx} \end{pmatrix}$$

where

$$\hat{\Sigma}_{yy} = \hat{\Lambda}_y \hat{C} \hat{\Lambda}_y' + \hat{\Theta}_\epsilon^2$$

$$\hat{\Sigma}_{xy} = \hat{\Lambda}_x \hat{\Phi} \hat{D}' \hat{\Lambda}_y'$$

$$\hat{\Sigma}_{xx} = \hat{\Lambda}_x \hat{\Phi} \hat{\Lambda}_x' + \hat{\Theta}_\delta^2$$

$$\hat{C} = \hat{D}\hat{\Phi}\hat{D}' + \hat{B}^{-1}\hat{\Psi}\hat{B}^{-1'}$$

$$\hat{D} = \hat{B}^{-1}\hat{\Gamma}$$

If the fit is good $\hat{\Sigma}$ should agree well with S and the residual matrix, $\hat{\Sigma} - S$, should be small. Elements of the residual matrices may suggest how the hypothesized structure should be modified to obtain a better fit. The matrices are printed row-wise, each element with three decimals.

4.4. Standardized Solution

If column 47 or the parameter card is T (see 3.2), the standardized solution ($\hat{\Lambda}_y^*$, $\hat{\Lambda}_x^*$, $\hat{B}^*$, $\hat{\Gamma}^*$, $\hat{\Phi}^*$ and $\hat{\Psi}^*$) will be printed, as well as the standardized matrices $\hat{C}^*$ and $\hat{D}^*$. That is:

$$\hat{\Lambda}^*_y = \hat{\Lambda}_y \hat{A}_\eta \quad ,$$

$$\hat{\Lambda}^*_x = \hat{\Lambda}_x \hat{A}_\xi \quad ,$$

$$\hat{B}^* = \hat{A}_\eta^{-1} \hat{B} \hat{A}_\eta \quad ,$$

$$\hat{\Gamma}^* = \hat{A}_\eta^{-1} \hat{\Gamma} \hat{A}_\xi \quad ,$$

$$\hat{\Phi}^* = \hat{A}_\xi^{-1} \hat{\Phi} \hat{A}_\xi^{-1} \quad ,$$

$$\hat{\Psi}^* = \hat{A}_\eta^{-1} \hat{\Psi} \hat{A}_\eta^{-1} \quad ,$$

$$\hat{D}^* = \hat{A}_\eta^{-1} \hat{D} \hat{A}_\xi \quad ,$$

$$\hat{C}^* = \hat{A}_\eta^{-1} \hat{C} \hat{A}_\eta^{-1} \quad ,$$

where

$$\hat{A}_\eta = (\text{diag } \hat{C})^{1/2} \quad ,$$

$$\hat{A}_\xi = (\text{diag } \hat{\Phi})^{1/2} \quad .$$

4.5. Technical Output

If column 53 of the parameter card is 4, 5 or 7 (see 3.2), the technical output is printed. This consists of a series of tables which describe the behavior of the iterative procedure and give various measures of the accuracy of the final solution. Ordinary users will have little interest in these tables.

The tables show the behavior of the iterative procedure under the steepest descent iterations and under the following iterations by the Davidon-Fletcher-Powell method. For interpretation of these tables the reader is referred to Gruvaeus and Jöreskog (1970). If something goes wrong, so that IND is 1, 2 or 3 (see 4.1), these tables may contain valuable information.

4.6. Punched Output

If column 45 of the parameter card is T (see 3.2), the final solution is punched on cards. The matrices are punched on cards in vector form, reading row-wise, each preceded by a format card. Only the lower diagonal parts of $\underset{\sim}{\Phi}$ and $\underset{\sim}{\Psi}$ will be punched. If column 41 of the parameter card is F, $\underset{\sim}{\Lambda}_x$, $\underset{\sim}{\Phi}$, $\underset{\sim}{\Theta}_\delta$ and $\underset{\sim}{\Gamma}$ will not be punched. If column 42 of the parameter card is F, $\underset{\sim}{\Psi}$ will not be punched. And if column 43 of the parameter card is F, $\underset{\sim}{B}$ will not be punched.

If IND $\neq$ 0 (see 4.1), the final solution will be automatically punched, regardless of the value of column 45 on the parameter card.

References

Blalock, H. M., Jr. Multiple indicators and the causal approach to measurement error. American Journal of Sociology, 1969, 75, 264-272.

Blalock, H. M., Jr. Causal models in the social sciences. Chicago: Aldine-Atherton, 1971.

Costner, H. L. Theory, deduction, and rules of correspondence. American Journal of Sociology, 1969, 75, 245-263.

Duncan, O. D. Path analysis: Sociological examples. American Journal of Sociology, 1966, 72, 1-16.

Duncan, O. D., Haller, A. O., & Portes, A. Peer influences on aspirations: A reinterpretation. American Journal of Sociology, 1968, 74, 119-137.

Fletcher, R., & Powell, M. J. D. A rapidly convergent descent method for minimization. Computer Journal, 1963, 6, 163-168.

Geraci, V. J., & Goldberger, A. S. Simultaneity and measurement error. Social Systems Reseach Institute Workshop Paper EME 7026. Madison, Wisc.: University of Wisconsin, 1971.

Goldberger, A. S. Econometric theory. New York: Wiley, 1964.

Gruvaeus, G. T., & Jöreskog, K. G. A computer program for minimizing a function of several variables. Research Bulletin 70-14. Princeton, N.J.: Educational Testing Service, 1970.

Hauser, R. M., & Goldberger, A. S. The treatment of unobservable variables in path analysis. In H. L. Costner (Ed.), Sociological methodology 1971. San Francisco: Jossey-Bass, 1971. Pp. 81-117.

Heise, D. R. Problems in path analysis and causal inference. In E. F. Borgatta (Ed.), Sociological methodology 1969. San Francisco: Jossey-Bass, 1970. Pp. 3-27.

Johnston, J. J. Econometric methods. (2nd ed.) New York: McGraw-Hill, 1972.

Jöreskog, K. G. A general approach to confirmatory maximum likelihood factor analysis. Psychometrika, 1969, 34, 183-202.

Jöreskog, K. G. A general method for analysis of covariance structures. Biometrika, 1970, 57, 239-251.

Jöreskog, K. G. A general method for estimating a linear structural equation system. In A. S. Goldberger & O. D. Duncan (Eds.), Structural equation models in the social sciences. New York: Seminar Press, 1973, in press.

Land, K. C. Principles of path analysis. In E. F. Borgatta (Ed.), Sociological methodology 1969. San Francisco: Jossey-Bass, 1969. Pp. 3-37.

Malinvaud, E. Statistical methods of econometrics. (2nd ed.) Amsterdam: North-Holland, 1970.

Turner, M. E., & Stevens, C. D. The regression analysis of causal paths. Biometrics, 1959, 15, 236-258.

Wright, S. The method of path coefficients. Annals of Mathematical Statistics, 1934, 5, 161-215.

Wright, S. The interpretation of multivariate systems. In O. Kempthorne, et al. (Eds.), Statistics and mathematics in biology. Ames, Iowa: Iowa State University, 1954.

Wright, S. The treatment of reciprocal interaction, with or without lag, in path analysis. Biometrics, 1960, 16, 423-445.

Appendix

We shall illustrate how input data are set up and what the printout looks like by means of three small data sets. These data also serve as test data to be run when the program has been compiled on a different computer.

Pages A3 - A4 show card by card how the input data are punched. One line corresponds to one card. Pages A5 - A34 show the corresponding printout obtained.

The first set of data is the artificial data discussed in section 1.4. No intermediate output is requested and the standardized solution is not printed.

The second set of data is the model of Duncan, Haller and Portes discussed in section 1.5. There is no selection of variables from the matrix, but columns (rows) are to be reordered. The standardized solution is requested and printed. Note that the correct number of degrees of freedom is twelve and not 33 as given by the program. Since we know the solution $\hat{\Sigma} = S_{xx}$, we treated Φ as fixed at S_{xx} when the program was run. But Φ should be considered as free, which accounts for the discrepancy in degrees of freedom.

The third set of data is the restricted factor analysis model discussed in section 1.6. Note that this model neither Ψ nor B are to be read in. Columns (rows) of the input matrix are to be reordered. Intermediate output is requested.

For all three data sets both the input matrix and the matrix to be analyzed are correlation matrices.

At various places in the output, time estimates are printed. The time shown is the time taken to compute the solution that follows the time estimate. This time includes only the iterations and not the time for printing, except possibly the technical printout.

```
ARTIFICIAL DATA
   4     7     2     3   10          23       TTTEEEE   343
(8F10.0)
     3.815     3.195     3.[illegible]     3.89[illegible]    3.5[illegible]    5.773     4.259     3.842
     6.192     6.973      .764      .69      .176      .193     1.374      .994
      .896      .229      .251     1.298     1.952     1.69[illegible]   1.525      .347
      .38[illegible]    1.739     2.76[illegible]   4.224      .835      .753      .157      .172
       .7       .9[illegible]    2.[illegible]69    1.471      .917      .827      .173      .189
      .768      .998     2.272     1.317     1.822     1.259     1.136     1.824
     1.997      .6[illegible]1      .781      .9[illegible]2      .3       .33      1.675     1.763
     1.591     2.554     2.797      .841     1.094     1.262      .421      .452
     1.958     2.914
22221111
(80I1)
[illegible]01[illegible]01
(80I1)
[illegible]1
(80I1)
[illegible]110
(80I1)
320[illegible]1
40014[illegible]29
(16F5.0)
  1.   0.   .9   [illegible]   [illegible]   1.   0.   .9
(16F5.0)
  1.   0.   0.   1.1  [illegible]   0.   1.   1.1   0.   0.   1.   0.   0.   .9   0.   0.
  0.   1.   0.   0.   1.5
(16F5.0)
  1.  -.5  -.5   1.
(16F5.0)
  .2   .2   0.  -1.   [illegible]   1.3
(16F5.0)
  1.   .6   1.  -.5   .5   1.5
(16F5.0)
  .4   .4   .8
(16F5.0)
  .5   .5   .5   .5
(16F5.0)
  .5   .5   .5   .5   .5   .5   .5
DUNCAN,HALLER,PORTES DATA        -      PEER INFLUENCES ON ASPIRATIONS
   4     6     6    10   329        270      TTTEETT   343
(8F10.0)
       1.      .1839       1.      .2220      .0489       1.      .4105     .2137
     .3240       1.      .4043      .2742      .4047      .6247       1.      .3355
     .0782     .2302      .2995      .2863       1.      .1021      .1147     .0931
     .0760     .0702      .2087       1.      .1861      .0186      .2707     .293[illegible]
     .2407     .295[illegible]    -.0438       1.      .2598      .0839      .2786     .4216
     .3275     .5[illegible]7      .1989      .3667       1.      .2903      .1124     .3054
     .3269     .3669      .5191      .2784      .4105      .6404       1.
   4     6
   4     5    10     9     2     1     3     8     6     7
20220200
(80I1)
1[illegible]10[illegible]110[illegible]110[illegible]11010[illegible]10101
(80I1)
.1[illegible]00100[illegible]00[illegible]
(80I1)
1111[illegible]1111[illegible]
(80I1)
1[illegible]
```

```
(16F5.0)
 .538   0.   .5   0.    .   .5 .559   0.   0.   .5   .5   0.   0. .527   .5   0.
   .5   0.    . .465    .   .5   .    .5
(8,F1.0)
1....  0.1  ...  0.1 ... ..............  ..  ......
(8 F1.0)
10..0010000010.. 00010 .00010...01
(16F5.0)
 .324 .479 .391 .145   .   0.   0.   0.   0.   0.   0.   0. .129 .435 .677 .306
   0.   .   0.   .     .   0.   0.   0.   0.   0.   1.   0.   0.   0.   0.   0.
   0.   0.   0.   0.   0.   1.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
   1.   0.   0.   0.   .   0.   0.   0.   0.   0.   0.   1.
(8F1..0)
       1.     .1839       1.     .0489    .2220       1.     .0186     .1861
    .2707       1.     .0782     .3355    .2302     .2950       1.     .1147
    .1021     .0931    -.0438    .2087       1.        0.       0.        0.
       0.        0.       0.        1.       0.        0.       0.        0.
       0.        0.       0.        1.       0.        0.       0.        0.
       0.        0.       0.        0.       1.        0.       0.        0.
       0.        0.       0.        0.       0.        0.       1.
(16F5.0)
  1.-.175   1.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
   0.   0.   0.   0.   0.
(80F1.0)
00.0
(80F1.0)
.0000(
HOLZINGER-SWINEFORD RESTRICTED SOLUTION
 .  6   3   3   3  145        280      TFFFFTF   347
(8F10.0)
       1.     .3258       1.     .4486    .4170       1.     .3416     .2230
    .3279       1.     .3191     .1595    .2869     .7186       1.     .3171
    .1947     .3473    .7145     .6853       1.     .1042    .0664     .0746
    .2089     .2539    .1787        1.    .3076     .1680    .2386     .1038
    .1978     .1211    .5871        1.    .4868     .2479    .3726     .3144
    .3556     .2718    .4183     .5283       1.
    6   3
    2   3    5    6    8    9    1    4    7
22002011
(80I1)
1001000100100001001
(80I1)
100010001
(80I1)
010110
(16F5.0)
   .5   0.   0.   .6    .   0.    .   .8   0.   0.   .8   0.   0.   0.   .8   0.
   0.   .8
(16F5.0)
   .7   0.    .   0.   .8   0.   0.   0.   .6
(80F1.0)
100010001
(16F5.0)
   1.   .5   1.   .5   3   1.
(16F5.0)
   .9   .7   .5   .6   5   .7
(16F5.0)
   .6   .5   .8
STOP
```

```
LINEAR STRUCTURAL RELATIONSHIPS
   ARTIFICIAL DATA
     P =    4
     Q =    7
     M =    2
     N =    3
     NP=  1??
     LOGICAL INDICATORS= TTTFFFF
     INTEGER INDICATORS= 343
     ESTIMATED TIME IN SECONDS=  280.
```

SYY

	1	2	3	4
1	3.815			
2	3.125	3.068		
3	3.890	3.509	5.773	
4	4.259	3.842	6.192	6.973

SXY

	1	2	3	4
1	0.764	0.690	0.176	0.193
2	0.994	0.896	0.229	0.251
3	1.690	1.525	0.347	0.380
4	0.835	0.753	0.157	0.172
5	0.917	0.827	0.173	0.189
6	1.259	1.136	1.824	1.997
7	1.763	1.591	2.554	2.797

SXX

	1	2	3	4	5	6	7
1	1.374						
2	1.298	1.952					
3	1.739	2.260	4.224				
4	0.730	0.909	2.069	1.471			
5	0.768	0.998	2.272	1.317	1.822		
6	0.631	0.781	0.902	0.300	0.330	1.675	
7	0.841	1.094	1.262	0.421	0.462	1.958	2.914

PARAMETER SPECIFICATIONS

LAMBDA Y

1	0
0	0
0	2

LAMBDA X

0	0	0
3	0	0
4	5	0
0	0	0
0	6	0
0	0	0
0	0	7

BETA

0	8
9	0

GAMMA

10	10	0
0	0	11

PHI

12		
13	14	
15	16	17

PSI

18	
19	20

THETA EPS

21	22	23	24

THETA DELTA

25	26	27	28	29	30	31

INITIAL SOLUTION

LAMBDA Y

	1	2
1	1.000	0.0
2	0.900	0.0
3	0.0	1.000
4	0.0	0.900

LAMBDA X

	1	2	3
1	1.000	0.0	0.0
2	1.100	0.0	0.0
3	1.000	1.100	0.0
4	0.0	1.000	0.0
5	0.0	0.900	0.0
6	0.0	0.0	1.000
7	0.0	0.0	1.500

BETA

	1	2
1	1.000	-0.500
2	-0.500	1.000

GAMMA

	1	2	3
1	0.200	0.200	0.0
2	-1.000	0.0	1.300

PHI

	1	2	3
1	1.000		
2	0.600	1.000	
3	0.500	0.500	1.500

PSI

	1	2
1	0.400	
2	0.400	0.800

THETA EPS

	1	2	3	4
1	0.500	0.500	0.500	0.500

THETA DELTA

	1	2	3	4	5	6	7
1	0.500	0.500	0.500	0.500	0.500	0.500	0.500

TIME= 116.80

MAXIMUM LIKELIHOOD SOLUTION

IND = 1

LAMADA Y

	1	2
1	1.000	0.0
2	0.902	0.0
3	0.0	1.000
4	0.0	1.095

LAMADA X

	1	2	3
1	1.000	0.0	0.0
2	1.300	0.0	0.0
3	0.901	1.200	0.0
4	0.0	1.000	0.0
5	0.0	1.098	0.0
6	0.0	0.0	1.000
7	0.0	0.0	1.400

BETA

	1	2
1	1.000	-0.493
2	-0.595	1.000

GAMMA

	1	2	3
1	0.399	0.399	0.0
2	-1.000	0.0	1.198

PHI

	1	2	3
1	0.999		
2	0.699	1.199	
3	0.601	0.300	1.398

PSI

	1	2
1	0.506	
2	0.336	0.705

THETA EPS

	1	2	3	4
1	0.522	0.432	0.357	0.451

THETA DELTA

	1	2	3	4	5	6	7
1	0.613	0.514	0.419	0.521	0.613	0.526	0.416

REDUCED FORM MATRIX D = BETA INVERSE*GAMMA

	1	2	3
1	-0.133	0.565	0.837
2	-1.079	0.336	1.696

C = D*PHI*D' + BETA INVERSE*PSI*BETA INVERSE'

	1	2
1	3.543	
2	3.830	5.645

SIGMA(YY)

	1	2	3	4
1	3.815			
2	3.195	3.063		
3	3.890	3.508	5.773	
4	4.259	3.842	6.182	6.973

SIGMA(XY)

	1	2	3	4
1	0.764	0.689	0.176	0.193
2	0.994	0.896	0.229	0.251
3	1.691	1.525	0.347	0.380
4	0.835	0.753	0.157	0.172
5	0.917	0.827	0.173	0.189
6	1.259	1.136	1.824	1.997
7	1.763	1.590	2.554	2.797

SIGMA(XX)

	1	2	3	4	5	6	7
1	1.374						
2	1.298	1.952					
3	1.739	2.260	4.224				
4	0.699	0.909	2.069	1.471			
5	0.768	0.998	2.272	1.317	1.822		
6	0.601	0.781	0.902	0.300	0.330	1.675	
7	0.841	1.094	1.262	0.421	0.462	1.958	2.914

RESIDUALS = SIGMA - S

SIGMA(YY) - SYY

	1	2	3	4
1	0.000			
2	0.000	0.000		
3	-0.000	0.000	-0.000	
4	0.000	-0.000	0.000	0.000

SIGMA(XY) - SXY

	1	2	3	4
1	0.000	-0.001	0.000	-0.000
2	-0.000	0.000	-0.000	-0.000
3	0.001	-0.000	0.000	0.000
4	0.000	0.000	0.000	0.000
5	0.000	0.000	-0.000	0.000
6	0.000	-0.000	0.000	0.000
7	0.000	-0.001	-0.000	-0.000

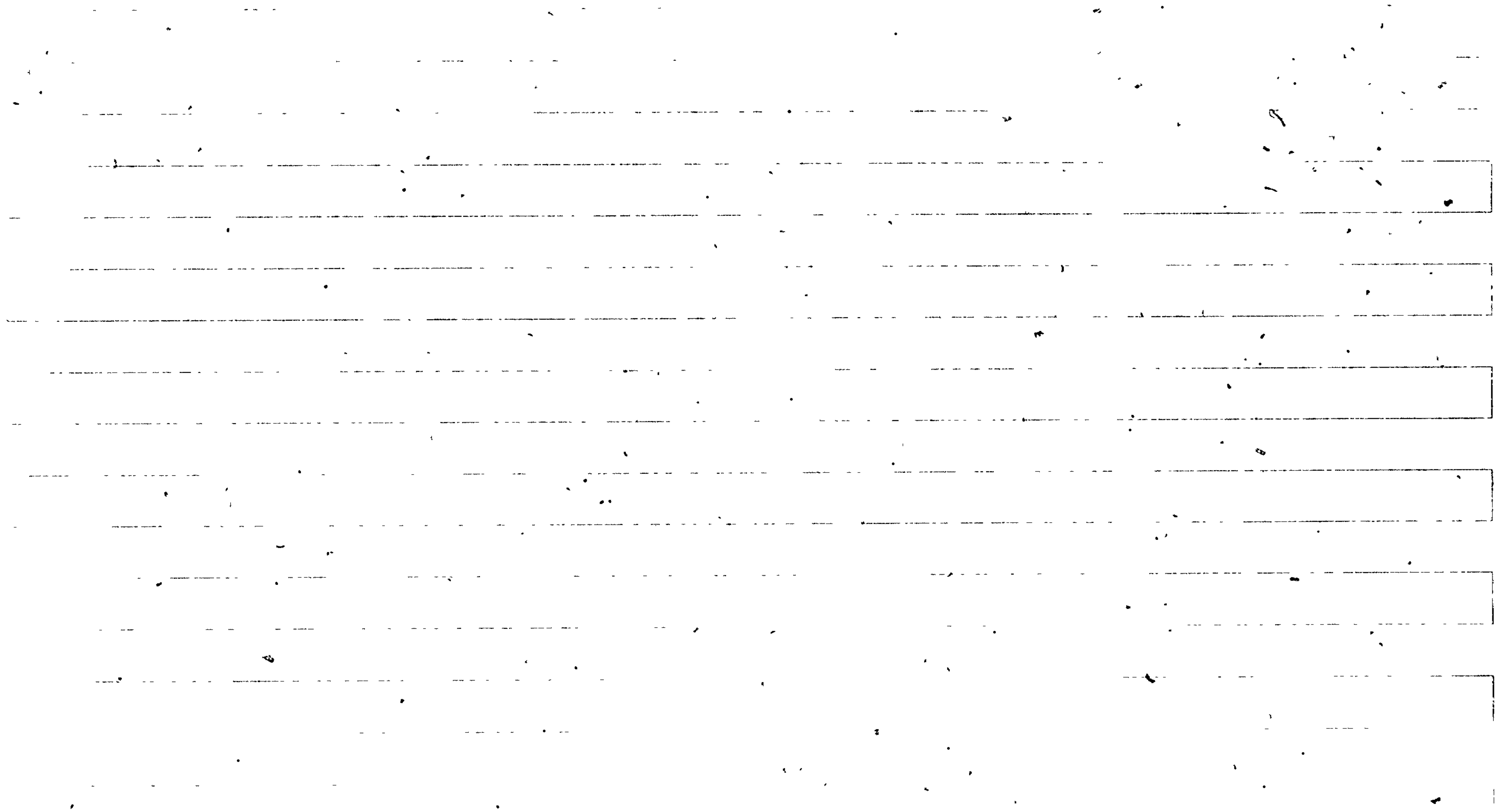
SIGMA(XX) - SXX

	1	2	3	4	5	6	7
1	0.000						
2	0.000	0.000					
3	-0.000	0.000	0.000				
4	-0.001	0.000	-0.000	-0.000			
5	0.000	0.000	0.000	-0.000	0.000		
6	-0.000	0.000	-0.000	0.000	-0.000	-0.000	
7	0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.000

TEST OF GOODNESS OF FIT

CHISQUARE WITH 35 DEGREES OF FREEDOM IS 0.0042

PROBABILITY LEVEL = 1.0000



LINEAR STRUCTURAL RELATIONSHIPS

DUNCAN,HALLER,PORTES DATA - PEER INFLUENCES ON ASPIRATIONS

P = 4

Q = 6

M = 6

N = 10

NP= 329

LOGICAL INDICATORS= TTTFFTT

INTEGER INDICATORS= 343

ESTIMATED TIME IN SECONDS= 2.70.

SYY

	1	2	3	4
1	1.000			
2	0.625	1.000		
3	0.327	0.367	1.000	
4	0.422	0.328	0.640	1.000

SXY

	1	2	3	4
1	0.214	0.274	0.112	0.084
2	0.411	0.404	0.290	0.260
3	0.324	0.405	0.305	0.279
4	0.293	0.241	0.411	0.361
5	0.300	0.286	0.519	0.501
6	0.076	0.070	0.278	0.199

SXX

	1	2	3	4	5	6
1	1.000					
2	0.184	1.000				
3	0.049	0.222	1.000			
4	0.019	0.186	0.271	1.000		
5	0.078	0.336	0.230	0.295	1.000	
6	0.115	0.102	0.093	-0.044	0.209	1.000

```
PARAMETER SPECIFICATION

 LAMBDA Y
    1   0   2   0   0   3
    4   0   0   5   6   0
    0   7   8   0   9   0
    0  10   0   0   0  11

 LAMBDA X
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0

 BETA
    0  12   0   0   0   0
   13   0   0   0   0   0
    0   0   0   0   0   0
    0   0   0   0   0   0
    0   0   0   0   0   0
    0   0   0   0   0   0

 GAMMA
   14  15  16  17   0   0   0   0   0   0
    0   0  18  19  20  21   0   0   0   0
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0

 PHI
    0
    0   0
    0   0   0
    0   0   0   0
    0   0   0   0   0
    0   0   0   0   0   0
    0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0

 PSI
    0
   22   0
    0   0   0
    0   0   0   0
    0   0   0   0   0
    0   0   0   0   0   0

 THETA EPS
    0   0   0   0

 THETA DELTA
    0   0   0   0   0   0
```

INITIAL SOLUTION

LAMBDA Y

	1	2	3	4	5	6
1	0.538	0.0	0.500	0.0	0.0	0.500
2	0.559	0.0	0.0	0.500	0.500	0.0
3	0.0	0.527	0.500	0.0	0.500	0.0
4	0.0	0.465	0.0	0.0	0.0	0.500

LAMBDA X

	1	2	3	4	5	6	7	8	9	10
1	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0

BETA

	1	2	3	4	5	6
1	1.000	0.0	0.0	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000

GAMMA

	1	2	3	4	5	6	7	8	9	10
1	0.304	0.478	0.391	0.145	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.129	0.435	0.677	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

PHI

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.184	1.000								
3	0.049	0.222	1.000							
4	0.019	0.186	0.271	1.000						
5	0.078	0.336	0.230	0.295	1.000					
6	0.115	0.102	0.093	-0.044	0.209	1.000				
7	0.0	0.0	0.0	0.0	0.0	0.0	1.000			
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000		
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

PSI

	1	2	3	4	5	6
1	1.000					
2	-0.175	1.000				
3	0.0	0.0	0.0			
4	0.0	0.0	0.0	0.0		
5	0.0	0.0	0.0	0.0	0.0	
6	0.0	0.0	0.0	0.0	0.0	0.0

THETA EPS

	1	2	3	4
1	0.0	0.0	0.0	0.0

THETA DELTA

	1	2	3	4	5	6
1	0.0	0.0	0.0	0.0	0.0	0.0

TIME= 72.90

MAXIMUM LIKELIHOOD SOLUTION

INF= 0

LAMBDA Y

	1	2	3	4	5	6
1	0.530	0.0	0.631	0.0	0.0	0.1[illegible]3
2	0.569	0.0	0.0	0.574	-0.003	0.0
3	0.0	0.530	0.067	0.0	0.531	0.0
4	0.0	0.472	0.0	0.0	0.0	0.652

LAMBDA X

	1	2	3	4	5	6	7	8	9	10
1	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0

BETA

	1	2	3	4	5	6
1	1.000	-0.177	0.0	0.0	0.0	0.0
2	-0.242	1.000	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000

GAMMA

	1	2	3	4	5	6	7	8	9	10
1	0.305	0.469	0.408	0.139	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.120	0.438	0.665	0.310	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

PHI

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.184	1.000								
3	0.049	0.222	1.000							
4	0.019	0.186	0.271	1.000						
5	0.078	0.336	0.230	0.295	1.000					
6	0.115	0.102	0.093	-0.044	0.209	1.000				
7	0.0	0.0	0.0	0.0	0.0	0.0	1.000			
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000		
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

PSI

	1	2	3	4	5	6
1	1.000					
2	-0.105	1.000				
3	0.0	0.0	0.0			
4	0.0	0.0	0.0	0.0		

5	0.0	0.0	0.0	0.0	0.0	
6	0.0	0.0	0.0	0.0	0.0	0.0

THETA EPS

	1	2	3	4
1	0.0	0.0	0.0	0.0

THETA DELTA

	1	2	3	4	5	6
1	0.0	0.0	0.0	0.0	0.0	0.0

REDUCED FORM MATRIX C = BETA INVERSE*GAMMA

	1	2	3	4	5	6	7	8	9	10
1	0.319	0.450	0.449	0.226	0.123	0.057	0.0	0.0	0.0	0.0
2	0.077	0.119	0.229	0.493	0.695	0.324	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

C = C*PHI*C' + BETA INVERSE*PSI*BETA INVERSE'

	1	2	3	4	5	6
1	2.069					
2	1.257	2.550				
3	0.0	0.0	1.000			
4	0.0	0.0	0.0	1.000		
5	0.0	0.0	0.0	0.0	1.000	
6	0.0	0.0	0.0	0.0	0.0	1.000

SIGMA(YY)

	1	2	3	4
1	1.000			
2	0.625	1.000		
3	0.311	0.378	1.003	
4	0.408	0.338	0.638	0.994

SIGMA(XY)

	1	2	3	4
1	0.239	0.257	0.112	0.099
2	0.391	0.420	0.287	0.256
3	0.354	0.380	0.309	0.275
4	0.254	0.272	0.408	0.363
5	0.262	0.281	0.533	0.475
6	0.107	0.115	0.260	0.231

SIGMA(XX)

	1	2	3	4	5	6
1	1.000					
2	0.184	1.000				
3	0.049	0.222	1.000			
4	0.019	0.186	0.271	1.000		
5	0.078	0.336	0.230	0.295	1.000	
6	0.115	0.102	0.093	-0.044	0.209	1.000

RESIDUALS = SIGMA - S

SIGMA(YY) - SYY

	1	2	3	4
1	0.000			
2	0.006	-0.000		
3	-0.016	0.011	0.003	
4	-0.014	0.010	-0.002	-0.006

SIGMA(XY) - SXY

	1	2	3	4
1	0.025	-0.018	-0.001	0.016
2	-0.020	0.015	-0.003	-0.004
3	0.030	-0.025	0.003	-0.004
4	-0.039	0.032	-0.003	0.002
5	-0.038	-0.005	0.014	-0.026
6	0.031	0.044	-0.019	0.032

SIGMA(XX) - SXX

	1	2	3	4	5	6
1	0.0					
2	0.0	0.0				
3	0.0	0.0	0.0			
4	0.0	0.0	0.0	0.0		
5	0.0	0.0	0.0	0.0	0.0	
6	0.0	0.0	0.0	0.0	0.0	0.0

STANDARDIZED SOLUTION

LAMBDA Y

	1	2	3	4	5	6
1	0.763	0.0	0.631	0.0	0.0	0.143
2	-0.819	0.0	0.0	0.574	-0.003	0.0
3	0.0	0.847	-0.067	0.0	0.531	0.0
4	0.0	0.754	0.0	0.0	0.0	0.652

LAMBDA X

	1	2	3	4	5	6	7	8	9	10
1	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0

BETA

	1	2	3	4	5	6
1	1.000	-0.196	0.0	0.0	0.0	0.0
2	-0.213	1.000	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000

GAMMA

	1	2	3	4	5	6	7	8	9	10
1	0.212	0.326	0.284	0.097	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.075	0.274	0.416	0.194	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

PHI

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.184	1.000								
3	0.049	0.222	1.000							
4	0.019	0.186	0.271	1.000						
5	0.078	0.336	0.230	0.295	1.000					
6	0.115	0.102	0.093	-0.044	0.209	1.000				
7	0.0	0.0	0.0	0.0	0.0	0.0	1.000			
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000		
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

PSI

	1	2	3	4	5	6
1	0.483					
2	-0.046	0.392				
3	0.0	0.0	0.0			
4	0.0	0.0	0.0	0.0		
5	0.0	0.0	0.0	0.0	0.0	
6	0.0	0.0	0.0	0.0	0.0	0.0

THETA EPS

	1	2	3	4
1	0.0	0.0	0.0	0.0

THETA DELTA

	1	2	3	4	5	6
1	0.0	0.0	0.0	0.0	0.0	0.0

D (STANDARDIZED)

	1	2	3	4	5	6	7	8	9	10
1	0.221	0.340	0.312	0.157	0.085	0.040	0.0	0.0	0.0	0.0
2	0.048	0.074	0.143	0.308	0.435	0.203	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

C (STANDARDIZED)

	1	2	3	4	5	6
1	1.000					
2	0.547	1.000				
3	0.0	0.0	1.000			
4	0.0	0.0	0.0	1.000		
5	0.0	0.0	0.0	0.0	1.000	
6	0.0	0.0	0.0	0.0	0.0	1.000

TEST OF GOODNESS OF FIT

CHISQUARE WITH 33 DEGREES OF FREEDOM IS 11.4327

PROBABILITY LEVEL = 0.9998

LINEAR STRUCTURAL RELATIONSHIPS

HOLZINGER-SWINEFORD RESTRICTED SOLUTION

P = 6

Q = 3

M = 3

N = 3

NP= 145

LOGICAL INDICATORS= TFFFFTF

INTEGER INDICATORS= 347

ESTIMATED TIME IN SECONDS= 280.

SYY

	1	2	3	4	5	6
1	1.0					
2	0.417	1.000				
3	0.169	0.287	1.00			
4	0.195	0.347	0.685	1.000		
5	0.168	0.239	0.198	0.121	1.000	
6	0.248	0.373	0.356	0.272	0.528	1.000

SXY

	1	2	3	4	5	6
1	0.326	0.449	0.309	0.317	0.308	0.487
2	0.228	0.328	0.719	0.715	0.104	0.314
3	0.066	0.075	0.254	0.179	0.587	0.418

SXX

	1	2	3
1	1.000		
2	0.342	1.000	
3	0.1?4	0.209	1.000

```
PARAMETER SPECIFICATIONS

    LAMBDA Y
       1   0   0
       2   0   0
       0   3   0
       0   4   0
       0   0   5
       0   0   6

    LAMBDA X
       7   0   0
       0   8   0
       0   0   9

    BETA
       0   0   0
       0   0   0
       0   0   0

    GAMMA
       0   0   0
       0   0   0
       0   0   0

    PHI
       0
      10   0
      11  12   0

    THETA EPS
      13  14  15  16  17  18

    THETA DELTA
      19  20  21
```

INITIAL SOLUTION

LAMBDA Y

	1	2	3
1	0.500	0.0	0.0
2	0.600	0.0	0.0
3	0.0	0.800	0.0
4	0.0	0.800	0.0
5	0.0	0.0	0.800
6	0.0	0.0	0.800

LAMBDA X

	1	2	3
1	0.700	0.0	0.0
2	0.0	0.800	0.0
3	0.0	0.0	0.600

BETA

	1	2	3
1	1.000	0.0	0.0
2	0.0	1.000	0.0
3	0.0	0.0	1.000

GAMMA

	1	2	3
1	1.000	0.0	0.0
2	0.0	1.000	0.0
3	0.0	0.0	1.000

PHI

	1	2	3
1	1.000		
2	0.500	1.000	
3	0.500	0.300	1.000

THETA EPS

	1	2	3	4	5	6
1	0.900	0.700	0.500	0.600	0.600	0.700

THETA DELTA

	1	2	3
1	0.600	0.500	0.800

HAVIOR UNDER STEEPEST DESCENT ITERATIONS

ITER	TRY	ABSCISSA	SLOPE	FUNCTION
1	0	0.0	-0.497457820 00	0.223341520 00
	1	0.100000000 00	-0.145653770 00	0.191816040 00
	2	0.149564950 00	0.291941850-02	0.188340520 00
2	0	0.0	-0.164757040 00	0.138340520 00
	1	0.149564950 00	0.239204420 00	0.192243250 00
	2	0.671160450-01	0.127325290-03	0.182681460 00
3	0	0.0	-0.112711380 00	0.182681460 00
	1	0.671160450-01	0.756[illegible]82240-01	0.181677000 00
	2	0.382910460-01	-0.379235940-04	0.180570320 00

BEHAVIOR UNDER FLEPOW ITERATIONS

ITER	TRY	ABSCISSA	SLOPE	FUNCTION
1	0	0.0	-0.72793393D-01	0.18057032D 00
	1	0.10000000D 00	0.19012295D 00	0.18578232D 00
	2	0.39874533D-01	0.19733796D-03	0.17943270D 00
2	0	0.0	-0.46122192D-01	0.17943270D 00
	1	0.39874533D-01	0.22556525D-01	0.17837181D 00
	2	0.21099590D-01	0.45057649D-05	0.17804148D 00
3	0	0.0	-0.21329839D-01	0.17804148D 00
	1	0.66109059D-01	0.58090259D-01	0.17914902D 00
	2	0.16103972D-01	0.56835952D-05	0.17786986D 00
4	0	0.0	-0.14138935D-01	0.17786986D 00
	1	0.16103072D-01	0.57736440D-02	0.17780274D 00
	2	0.11419409D-01	-0.76736687D-07	0.17778922D 00
5	0	0.0	-0.60586141D-02	0.17778922D 00
	1	0.11419409D-01	0.28388354D-02	0.17777081D 00
	2	0.77797811D-02	-0.18792569D-07	0.17776564D 00
6	0	0.0	-0.39683728D-02	0.17776564D 00
	1	0.77797811D-02	0.41055970D-02	0.17776618D 00
	2	0.38237784D-02	0.83484527D-06	0.17775806D 00
7	0	0.0	-0.24755907D-02	0.17775806D 00
	1	0.38237784D-02	0.98418075D-03	0.17775520D 00
	2	0.27360508D-02	-0.57763306D-06	0.17775467D 00
8	0	0.0	-0.19115452D-02	0.17775467D 00
	1	0.27360508D-02	-0.30116370D-03	0.17775164D 00
	2	0.32477302D-02	0.48873846D-06	0.17775156D 00
9	0	0.0	-0.12388934D-02	0.17775156D 00
	1	0.32477302D-02	0.23638545D-02	0.17775339D 00
	2	0.11168119D-02	-0.77578554D-06	0.17775087D 00
10	0	0.0	-0.56649490D-03	0.17775087D 00
	1	0.11168119D-02	0.73688642D-03	0.17775097D 00
	2	0.48540532D-03	-0.27599228D-08	0.17775073D 00
11	0	0.0	-0.24451309D-03	0.17775073D 00
	1	0.48540532D-03	0.34503296D-03	0.17775076D 00
	2	0.20132093D-03	0.18555605D-07	0.17775071D 00
12	0	0.0	-0.99921457D-04	0.17775071D 00
	1	0.20132093D-03	0.11567480D-03	0.17775071D 00
	2	0.93305333D-04	0.81642318D-09	0.17775071D 00

TIME= 8.20

MAXIMUM LIKELIHOOD SOLUTION

IND= 0

LAMBDA Y

	1	2	3
1	0.517	0.0	0.0
2	0.694	0.0	0.0
3	0.0	0.829	0.0
4	0.0	0.826	0.0
5	0.0	0.0	0.796
6	0.0	0.0	0.701

LAMBDA X

	1	2	3
1	0.677	0.0	0.0
2	0.0	0.866	0.0
3	0.0	0.0	0.659

BETA

	1	2	3
1	1.000	0.0	0.0
2	0.0	1.000	0.0
3	0.0	0.0	1.000

GAMMA

	1	2	3
1	1.000	0.0	0.0
2	0.0	1.000	0.0
3	0.0	0.0	1.000

PHI

	1	2	3
1	1.000		
2	0.541	1.000	
3	0.523	0.336	1.000

THETA EPS

	1	2	3	4	5	6
1	0.856	0.720	0.559	0.563	0.605	0.713

THETA DELTA

	1	2	3
1	0.736	0.501	0.752

REDUCED FORM MATRIX D = BETA INVERSE*GAMMA

	1	2	3
1	1.000	0.0	0.0
2	0.0	1.000	0.0
3	0.0	0.0	1.000

SIGMA(YY)

	1	2	3	4	5	6
1	1.000					
2	0.358	1.000				
3	0.232	0.311	1.000			
4	0.231	0.31[illegible]	0.685	1.000		
5	0.215	0.289	0.222	0.221	1.000	
6	0.189	0.254	0.195	0.195	0.558	1.000

SIGMA(XY)

	1	2	3	4	5	6
1	0.34[illegible]	0.469	0.303	0.302	0.282	0.248
2	0.242	0.325	0.718	0.715	0.232	0.2[illegible]4
3	0.178	0.239	0.184	0.183	0.525	0.462

SIGMA(XX)

	1	2	3
1	1.000		
2	0.317	1.000	
3	0.233	0.192	1.000

RESIDUALS = SIGMA - S

SIGMA(YY) - SYY

	1	2	3	4	5	6
1	-0.000					
2	-0.059	0.000				
3	0.072	0.024	0.000			
4	0.036	-0.037	0.000	0.000		
5	0.047	0.050	0.024	0.100	0.000	
6	-0.058	-0.118	-0.16[illegible]	-0.[illegible]77	0.029	0.000

SIGMA(XY) - SXY

	1	2	3	4	5	6
1	0.024	0.021	-0.006	-0.015	-0.026	-0.239
2	0.014	-0.003	-0.001	0.[illegible]01	0.128	-0.111
3	0.112	0.165	-0.070	0.004	-0.062	0.044

SIGMA(XX) - SXX

	1	2	3
1	0.000		
2	-0.025	0.000	
3	0.129	-0.017	0.000

TEST OF GOODNESS OF FIT

CHISQUARE WITH 24 DEGREES OF FREEDOM IS 51.1922

PROBABILITY LEVEL = 0.0010