On Electronic Voltage Stabilizers

F. V. Hunt and R. W. Hickman
Cruft Laboratory, Harvard University, Cambridge, Massachusetts
(Received September 2, 1938)

Voltage stabilizing circuits employing thermionic tubes can be classified into four groups, according to their derivation from (1) the transconductance bridge, (2) the amplification factor bridge, (3) the simple degenerative amplifier, and (4) combination circuits involving two or more of the foregoing classes or utilizing amplified control voltages. It is shown that the variational performance of a stabilizer is characterized by an internal output resistance and a stabilization ratio. The effect of source resistance on these performance parameters is discussed in the general case and expressions for these parameters are evaluated for ten stabilizer circuits, seven of which are original with the authors. All but two of the circuits can be adjusted to yield perfect stabilization and four have also a very low internal resistance. Glow discharge tubes are discussed as bias battery substitutes. A simple a.c. bridge technique which is convenient for the experimental determination of stabilizer performance under normal load conditions is described in an appendix.

Introduction

Several papers have appeared during the past few years dealing with vacuum tube circuits for the stabilization of the output voltage of low power rectifiers. These papers have been devoted principally to a discussion of the problem of voltage stabilization under constant load conditions. Many of our laboratory problems, on the other hand, have required circuits which would provide a power supply whose voltage is stabilized against the effect of a variable load, i.e., circuits having a low equivalent internal resistance. This need led us some time ago[1] to re-examine the work in this field from the point of view of stabilization with respect to variations of both load resistance and input voltage, and to undertake a systematic classification of the available stabilizer circuits according to their synthesis from a few elementary circuits. As a by-product of the systematic classification, several new circuits are here proposed which may have special advantages in some applications.

The analysis of the stabilization of the output voltage under variable load conditions requires the consideration of two factors which are unimportant for stabilizer circuits operating with a constant load. Assuming, for example, a constant input (line) voltage for the rectifier, one must in practice take into account not only the variation in output voltage produced by the varying load current flowing through the equivalent internal resistance of the stabilizing network, but also the effect of the change in input voltage to the stabilizer arising from the varying load current flowing through the equivalent internal resistance of the rectifier. In order to provide a common basis for the comparison of different stabilizing circuits, we proceed first to show that two parameters, the stabilization ratio S_0 and the internal resistance R_0, are sufficient to characterize the variational performance of the stabilizer when it is operated from a generator having zero internal resistance. A similar analysis shows that two corresponding parameters, S and R, are sufficient to characterize the over-all performance of a stabilizing circuit together with its associated rectifier. A relationship between these parameters is then established which allows the computation of S and R in terms of S_0, R_0, the rectifier resistance R_r, and small correction terms (usually negligible) involving other circuit constants. With the effect of the resistance of the source thus accounted for in the general case, the remainder of the paper is devoted to a study of the stabilization and internal resistance characteristics of various circuits considered as operating from resistanceless sources.

General Equivalent Circuits

The possibility of establishing a generalized equivalent circuit for voltage stabilizers arises

[1] J. Acous. Soc. Am. 7, 237(A) (1936); 8, 34 (1936). Phys. Rev. 50, 1094(A) (1936); 53, 913(A) (1938).

from the experimental observation that, within their useful range, all of the stabilizing circuits exhibit a nearly linear variation of output voltage with either input voltage or output current. On the basis of this linearity one may conclude that Thévenin's theorem may be applied and that, so far as the output terminals are concerned, the stabilizer circuit may be replaced by an electromotive force acting in series with an internal resistance, as illustrated in the upper part of Fig. 1. The electromotive force E', is equivalent to the open-circuit voltage across the output terminals and will be a linear function of the input voltage to the stabilizer, independent of the externally connected load. Our own and previous work on this subject has indicated that it is more convenient to define the stabilization ratio in terms of the input and output *terminal* voltages rather than in terms of the input voltage and the internal electromotive force, E'.

Our first objective, therefore, is to evaluate the open-circuit voltage, E', of the equivalent circuit in terms of the input voltage E_i, the internal resistance R_0, and a stabilization ratio S_0 defined in terms of the terminal voltages. Such a definition of the stabilization ratio always leads to formulas for S_0 that involve the load resistance R_L, so that we may expect the evaluation of E' in these terms to include R_L also, but in such a way that E' itself is independent of R_L as demanded by Thévenin's theorem. This is exhibited in the following analysis.

In considering the variation of output voltage with changes in the load, a convenient separation of variables is afforded by the introduction of an independent test voltage, e, in series with the load resistance, as indicated in Fig. 1. Considering e and E_i as the independent variables, we may now define the stabilization ratio, S_0, and the internal resistance, R_0, through the following differential quotients:

$$(\partial E_0/\partial E_i)_e = 1/S_0, \tag{1}$$

$$(\partial i_0/\partial e)_{E_i} = 1/(R_0+R_L). \tag{2}$$

Proceeding with the evaluation of E' in the equivalent circuit, Eq. (1) may be integrated to give

$$E_0 = (E_i + f_1(e))/S_0. \tag{3}$$

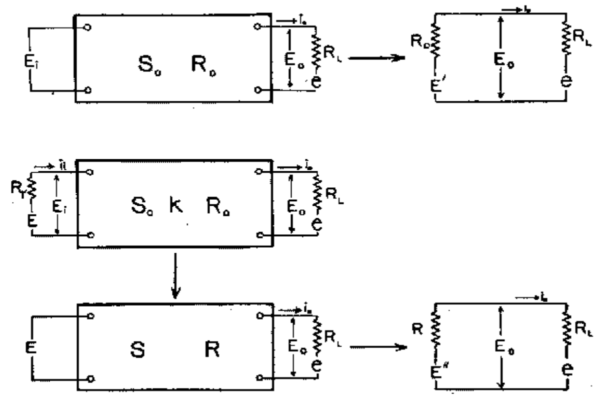


Fig. 1. Equivalent circuits and notation for the analysis of the effect of source resistance on stabilizer performance.

Differentiating (3) with respect to e gives

$$\partial E_0/\partial e = (1/S_0)(\partial f_1(e)/\partial e). \tag{4}$$

The left-hand side of (4) may also be evaluated from the Ohm's law equation for the output circuit,

$$E_0 = i_0 R_L - e \tag{5}$$

and allows the introduction of the defining equation for R_0. Thus

$$\frac{\partial E_0}{\partial e} = R_L \frac{\partial i_0}{\partial e} - 1 = -\frac{R_0}{R_0+R_L}. \tag{6}$$

Equating (4) and (6) and integrating gives

$$f_1(e) = -\frac{S_0 R_0}{R_0+R_L} e + K_1, \tag{7}$$

where K_1 is a constant. Putting (7) into (3) yields

$$E_0 = \frac{E_i+K_1}{S_0} - \frac{R_0}{R_0+R_L} e. \tag{8}$$

By eliminating e from (8) with the help of (5), the expression for E_0 may be put in convenient form for comparison with the upper equivalent circuit of Fig. 1.

$$\begin{aligned} E_0 &= \frac{(E_i+K_1)}{S_0/(1+R_0/R_L)} - i_0 R_0 \\ &= E' - i_0 R_0. \end{aligned} \tag{9}$$

As mentioned above, while S_0 is a function of R_L, the expression $S_0/(1+R_0/R_L)$ is not a func-

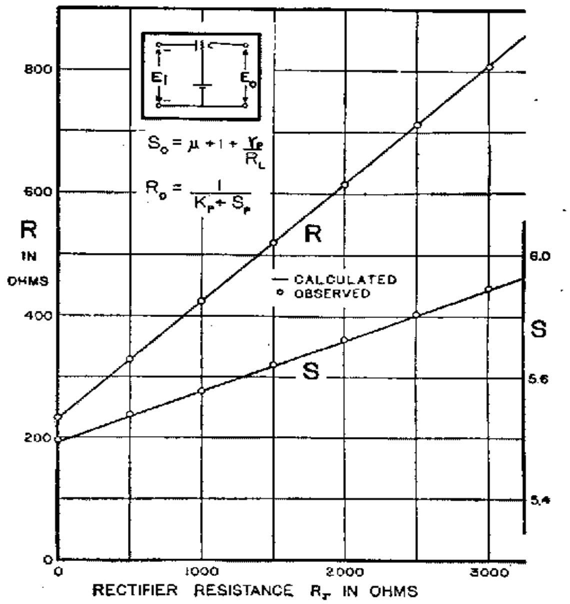


FIG. 2. Data verifying the predicted variation in the performance of a simple stabilizer when the internal resistance of the source is altered.

tion of R_L; it may in fact be shown to be equivalent to a different stabilization ratio defined as $(\partial E'/\partial E_i)$, and since E' is not a function of R_L by Thévenin's theorem, then the denominator of (9) is likewise independent of R_L. Eqs. (8) and (9) support the statement that variations in the output voltage arising from independent variations of input voltage or output current are indeed characterized by the two stabilizer parameters S_0 and R_0. These quantities are therefore characteristic of the configuration of the stabilizer itself and may be computed once and for all for each stabilizer circuit.

In practice the input for the stabilizer is usually derived from a source (hereafter called the rectifier) which itself has some internal resistance. This is illustrated in the central portion of Fig. 1. Such a circuit also exhibits a nearly linear variation of output voltage with either output current or rectifier e.m.f., E, the latter quantity being in turn linearly related to the a.c. line voltage.

We can then, as before, apply Thévenin's theorem to establish the equivalent circuit at the bottom of Fig. 1, and proceed to evaluate E'' in terms of the rectifier e.m.f., E, the over-all stabilization ratio, S, and the over-all internal resistance, R. The latter quantities are defined through the equations

$$(\partial E_0/\partial E)_e = 1/S, \tag{10}$$

$$(\partial i_0/\partial e)_E = 1/(R+R_L). \tag{11}$$

In carrying through a series of manipulations similar to those above, it is found necessary to take account of the fact that some of the input current to the stabilizer may be diverted from the output by the stabilizer circuit. Calling the current so diverted i_s, we have $i_i = i_0 + i_s$. This current, i_s, might be expressed in terms of either the input or the output voltage since they are themselves linearly related. We choose the input voltage for convenience and define an equivalent bleeder conductance k such that $i_s = k(E_i + K_2)$, where K_2 is another constant. Operating upon these equations as before leads to the following expression for the output voltage:

$$E_0 = \frac{(E+K_3)}{S/(1+R/R_L)} - i_0 R = E'' - i_0 R. \tag{12}$$

K_3 is again a constant of integration, and, as before, the denominator is independent of R_L in accordance with Thévenin's theorem.

If we now adjust E and R_r to produce the same stabilizer input voltage E_i used for the previous case everything to the right of E_i in the diagram is unchanged and we may equate the output voltages. This adjustment is made analytically by eliminating E_i from Eqs. (9) and (12) with the help of the Ohm's law equation for the input circuit, $E_i = E - i_i R_r$. Carrying out this elimination leads to the following expression:

$$E\left\{\frac{1}{\alpha S_0} - \frac{1}{S}\left(1 + \frac{R_r}{\alpha S_0 R_L}\right)\right\} - e\left\{\frac{R}{R+R_L} - \frac{R_0}{R_0+R_L} - \frac{R_r}{\alpha S_0(R+R_L)}\right\} + \left\{\frac{K_1}{S_0} - \frac{R_r k K_2}{\alpha S_0} - \frac{K_3}{S}\left(1 + \frac{R_r}{\alpha S_0 R_L}\right)\right\} = 0, \tag{13}$$

in which α is an abbreviation for $(1+kR_r)$. In order that this expression vanish for independent variations of E and e, each bracket must vanish individually. The first bracket yields the desired connection between S and S_0 as follows:

$$S = S_0(1+kR_r)+R_r/R_L, \tag{14}$$

while the second bracket gives the connection between R and R_0,

$$R = R_0 + \frac{R_r}{S_0}\frac{(1+R_0/R_L)}{(1+kR_r)}. \tag{15}$$

It will be noticed that the expression for R involves again the term $S_0/(1+R_0/R_L)$ which was shown above to be independent of R_L. It is to be expected that S will contain R_L, as does S_0, but it can be shown that the relation *between* S and S_0 is not a function of R_L by eliminating R_L from (14) with the help of the third bracket of (13). However, the equation for S which is obtained then contains the three constants which otherwise would not need to be evaluated in considering variational performance. Since the term in (14) involving R_L is always small for useful stabilizers, we have found it more convenient to use the forms given by (14) and (15). These expressions allow the effect of the rectifier resistance on the over-all performance of any stabilizer to be predicted. It may be pointed out that, in general, an increase in the rectifier resistance improves the voltage stabilization but increases the internal resistance of the stabilizer. On the other hand, if the stabilization factor is large, the rectifier resistance may be made fairly large without seriously affecting the internal resistance, thus allowing considerable economy in rectifier design. Experimental confirmation of these general relations is exhibited in Fig. 2 which shows S and R for a simple stabilizer (see below) plotted as functions of the rectifier resistance. The solid curves were calculated from the circuit constants and the measured tube coefficients, using the expressions given below for this particular stabilizer and Eqs. (14) and (15). The circles represent directly measured values. The experimental technique involved is discussed below in an appendix.

Classification of Stabilizing Circuits

All of the electronic stabilizers which we have found in the literature of the subject can be classified into the following categories:

(1) Circuits derived from the bridge circuit for measuring the variational transconductance of triodes. (*s circuits*)

(2) Circuits derived from the bridge circuit for measuring the variational amplification factor of triodes. (*u circuits*)

(3) Circuits derived from the single-stage degenerative d.c. amplifier. (*D circuits*)

(4) Combination circuits built up from the foregoing classes, or circuits formed by adding one or more stages of amplification to one of the foregoing classes.

1. *s* circuits

General.—An examination of the definition of variational transconductance will indicate the manner in which a bridge circuit for its measurement may be used as a stabilizer. This definition is $s_m = (di_p/de_g)$ with e_p held constant. The objective of a bridge circuit to measure this quantity is, therefore, to produce such variations of e_g and i_p that e_p *will* remain constant, and if the variations are produced at an audible frequency the constancy of e_p can be detected by a null balance with telephones. For our present purpose if the test voltage for the bridge circuit is supplied from a rectifier, and if e_p is taken as the output voltage of the device, then this output voltage will be perfectly stabilized over as wide a range of voltage or current as that for which s_m is sensibly constant. Such a circuit is shown in Fig. 3(a); it was proposed by King[2] in 1923 and reinvented by Kohler[3] in 1934.

Design and performance.—The bridge circuit represented by Fig. 3(a) is that designated as Fig. 4 in our paper[4] on the measurement of

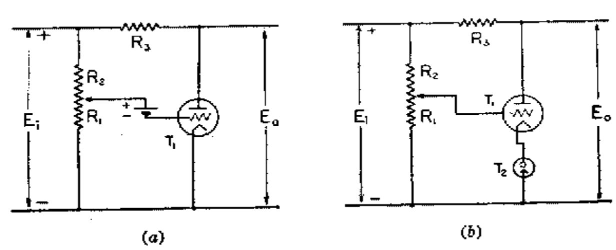


Fig. 3. Two modifications of the *s* type stabilizer derived from the basic transconductance bridge circuit.

[2] King, Bell Sys. Tech. J. **2**, 98 (1923).
[3] Kohler, Electronics, **7**, 388 (1934).
[4] Hickman and Hunt, Rev. Sci. Inst. **6**, 268 (1935).

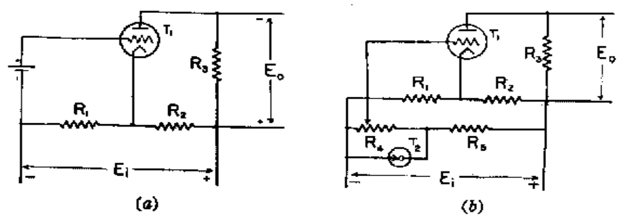


FIG. 4. Two modifications of the *u* type stabilizer derived from the basic amplification-factor bridge circuit.

electron tube coefficients, modified by the addition of the voltage dividing network R_1, R_2. The condition of balance for the stabilizer circuit is accordingly

$$s_m = 1/cR_3, \quad \text{where} \quad c = R_1/(R_1+R_2).$$

The stabilization ratio is infinite at balance, but the quantity for which the circuit is balanced, s_m, is usually a function of plate current so that the bridge will not remain in balance over a wide range of input voltage or output current. The internal resistance consists of the parallel combination of R_3 and the variational plate resistance of the tube, and is moderately high. Alternatively R_3 may be eliminated, by using the condition of balance, giving the internal resistance as the parallel combination of the variational plate resistance, and the grid-plate transfer resistance (=reciprocal of the transconductance) multiplied by $1/c$. To obtain a low internal resistance for this stabilizer one should therefore choose a low mu triode for T_1, having as large a transconductance as possible. Since both load current and tube current must flow through R_3, it is desirable to keep R_3 low, and this in turn requires the voltage divider ratio, c, to be large. Increasing the ratio c necessitates an increase in the bias battery and in the limit, as the divider is removed and the grid is returned to the positive input terminal, the bias battery must be greater than the output voltage by the negative grid bias required. An economic compromise must therefore be made in designing the circuit. One such compromise is illustrated by Fig. 3(b) in which a glow discharge lamp T_2 in the cathode circuit eliminates the bias battery. Using a receiving type triode and the 874 glow lamp, Fig. 3(b) is a useful stabilizer for constant-load service up to 30–40 ma and 100–400 volts. This type of bridge circuit offers the advantage

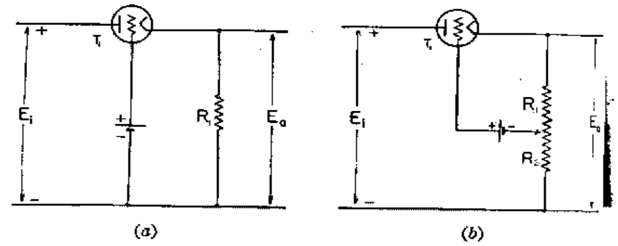


FIG. 5. Two modifications of the basic degenerative type stabilizer circuit.

of having the negative side of both input and output common.

2. *u* circuits

General.—A bridge circuit for measurement of the variational amplification factor,

$$u = -(de_p/de_g)$$

with i_p constant, is one arranged to apply small voltage variations to both grid and plate in such a ratio that the plate current remains constant. If the voltages applied to grid and plate are derived from a rectifier, and if the plate current flows through a constant resistance load circuit then the voltage drop across the load will be perfectly stabilized against small variations in the rectifier voltage. Such a circuit is shown in Fig. 4(a) and was proposed as a voltage stabilizing circuit by Street and Johnson[5] in 1932. Essentially the same circuit is discussed by Evans[6] who substitutes a pentode for the triode T_1, while Gingrich[7] has described the modification shown in Fig. 4(b) in which the bias battery is replaced by a glow discharge tube. The stabilizing circuit described by Ashworth and Mouzon[8] also falls in this category, although they have provided in addition some stabilization against changes in heater voltage.

Design and performance.—The bridge circuit shown in Fig. 4(a) is that designated as Fig. 9 in the reference cited[4] and the approximate condition of balance is simply $u = R_2/R_1$. The stabilization ratio is infinite at balance and, since the amplification factor is primarily determined by the tube geometry, the circuit will remain in approximate balance for reasonable variations of

[5] Street and Johnson, J. Frank. Inst. **214**, 155 (1932).
[6] Evans, Rev. Sci. Inst. **5**, 371 (1934).
[7] Gingrich, Rev. Sci. Inst. **7**, 207 (1936).
[8] Ashworth and Mouzon, Rev. Sci. Inst. **8**, 127 (1937).

input voltage. The circuit is balanced for constant output current so that small variations in load resistance are reflected as nearly proportional changes in output voltage. The internal resistance is high, being given by the series combination of R_2 and the variational plate resistance of the tube. The positive sides of rectifier and output are common so that this circuit is recommended when the positive side of the output circuit is to be grounded. The circuit is excellently adapted for low current, high voltage applications since only a portion of the output voltage appears across the stabilizing tube; thus receiving type tubes can be used to stabilize an output of 2000–3000 volts without exceeding their normal voltage ratings. For applications requiring several milliamperes of load current, W. N. Tuttle[9] in an unpublished study has considered the effect of the tube parameters on the over-all circuit efficiency. He finds that for a given output voltage, current and transconductance there will be an optimum value of u for minimum circuit losses. He found, for example, that for a load of 30 milliamperes at 180 volts and a tube whose transconductance was 2000 micromhos, u should be approximately ten for maximum efficiency.

3. *D* circuits

General.—In its simplest form the degenerative stabilizer is illustrated in Fig. 5(a) and consists of the plate-cathode circuit of a triode connected in series between the rectifier and its load, with the control grid returned to the common negative terminal through a bias battery. The operation of the circuit may be described as that of a series rheostat which is automatically controlled by the output voltage in such a direction as to tend to reduce the variations in output voltage. In the simplified form the grid biasing battery is required to be almost as large as the output voltage. A modification which reduces the battery voltage required is shown in Fig. 5(b) and was proposed by Street and Johnson[5] in 1932.

Design and performance.—The stabilization ratio for the circuit of Fig. 5(a) is approximately the amplification factor of the stabilizer tube, while the internal resistance is given by the parallel combination of the variational plate resistance and the grid-plate transfer resistance. Because of its simplicity this circuit was chosen for the equivalent circuit tests shown in Fig. 2. Although it is possible to modify this circuit to permit the use of a glow discharge tube instead of a battery for grid bias, the limited performance of the circuit renders its chief function that of serving as a foundation for the development of the combination circuits to be discussed below. In the modification of Fig. 5(b), if the fraction of the output voltage appearing between the grid tap and the positive output terminal is taken as $c(=R_1/(R_1+R_2))$, then the approximate stabilization ratio is reduced to cu while the internal resistance is increased to $R_0=1/(k_p+cs_m)$. An important property of this class of circuit is that the output voltage can be varied continuously by shifting the grid return tap, the output remaining stabilized without further adjustment.

4. Combination circuits

General.—In discussing the foregoing basic circuits, some comments have been made about embarrassing bias battery requirements and limitations of performance. It is natural to seek to remove these limitations by combinations of

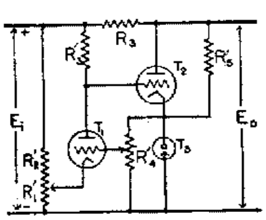


FIG. 6. *s* type stabilizer circuit with a *u* circuit arranged to provide a stabilized bias voltage.

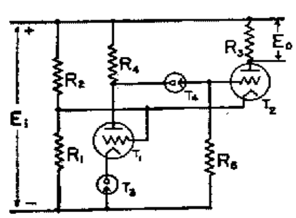
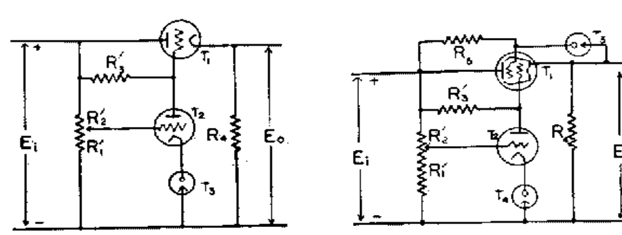


FIG. 7. *u* type stabilizer circuit arranged so that amplification of the control grid voltage allows a favorable selection of the resistor R_1.



(a) (b)

FIG. 8. Two modifications of the simple degenerative stabilizer employing an *s* circuit to provide a stabilized bias voltage; with proper adjustment these circuits simulate the bridge circuit for measurement of negative amplification factor.

[9] General Radio Company, Cambridge, Massachusetts.

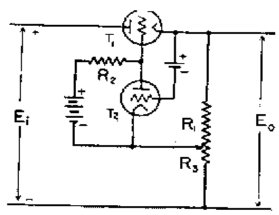


FIG. 9. Degenerative stabilizer provided with a separately-powered amplifier for the control voltage.

these basic circuits and to investigate the utility of providing additional amplification of the control voltages. Some of the following circuits were developed independently while the rest have been suggested by the systematic classification.

The first six of these circuits are especially useful for high voltage applications; the last is an extremely versatile circuit which has completely satisfied the laboratory demands which initiated this study.

A.—The circuit illustrated in Fig. 6 represents an ***s*** circuit with a ***u*** circuit arranged to provide the biasing voltage. The left-hand portion of the circuit, involving T_1 and the primed resistors, is exactly like the circuit of Fig. 4(b) and provides a perfectly stabilized voltage across R_3' when R_2'/R_1' is equal to the amplification factor of T_1. The remainder of the circuit is equivalent to that of Fig. 3(b). Since the drop across R_3' is stabilized, the grid of T_2 is effectively connected to the positive input terminal so far as variations are concerned. Thus, the output, or ***s*** portion of the circuit, is in balance when $R_3 = 1/s_{m2}$. Referring to Section 1 above, this is seen to reduce the internal resistance to its minimum value given by $R_0 = (k_{p2} + s_{m2})^{-1}$. A single glow discharge lamp T_3 provides the fixed bias for both portions of the circuit. The most useful feature of this circuit is its adaptability to high voltage, constant-load applications in which the negative side is to be grounded. It should be pointed out again that since s_m varies rapidly with changes in plate current, the condition of balance will be satisfied only for small variations of input voltage. The stabilizer tube T_2 must be rated for the full output voltage.

B.—For high voltage applications allowing the positive side to be grounded we have recommended the ***u*** circuit of Fig. 4(b). In this circuit the series resistance of R_1 and R_2 shunts the rectifier and should be kept high to avoid power loss. However, the entire load current flows through R_1 and this resistance should be kept low to avoid excessive loss of voltage. If these two opposing requirements are met by selecting a high mu tube, the plate resistance and the internal resistance of the stabilizer will be very high. Fig. 7 offers a solution of this dilemma by interposing a stage of amplification between the drop across R_1 and its application between the control grid and cathode of the tube T_2 for whose amplification factor the circuit is balanced. The value of R_1 required for balance may be reduced approximately in the ratio of the voltage gain of the amplifier stage (cf. Table II below) without otherwise altering the design or performance of the basic ***u*** circuit. A low mu tube may therefore be selected for T_2, keeping the internal resistance of the stabilizer low, and the circuit may be designed for as high output voltages as desired so long as the expected *variations* in output voltage do not exceed the voltage rating of T_2. An extra glow discharge lamp, T_4, is required for this circuit in order to secure the proper distribution of d.c. potentials on the grids of the two tubes. A study of the voltage distribution for both extremes of expected input voltage variation must be made in designing the circuit for specified conditions to determine how many lamps should be used in each indicated position.

The precaution of bringing the cathodes to operating temperature before the high voltage is applied should be observed in working with *all* circuits in which the tubes are rated for the *variations* in input voltage. Such procedure will prevent the entire voltage from appearing across the stabilizer tube with probable disastrous results.

C.—The two preceding circuits have been especially adapted for constant-load service. For applications in which the load is variable, a stabilizer of the degenerative type with its low internal resistance is desirable. Fig. 8(a) shows such a circuit in which an ***s*** circuit, as illustrated in Fig. 3(b), is used to provide the fixed bias indicated in the simple circuit of Fig. 5(a). A high internal resistance for the ***s*** portion of the circuit is not material in this case, since no current is drawn from it, so that T_2 may be

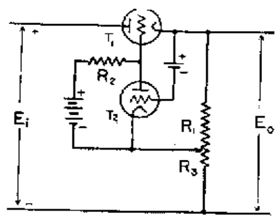


FIG. 9. Degenerative stabilizer provided with a separately-powered amplifier for the control voltage.

these basic circuits and to investigate the utility of providing additional amplification of the control voltages. Some of the following circuits were developed independently while the rest have been suggested by the systematic classification.

The first six of these circuits are especially useful for high voltage applications; the last is an extremely versatile circuit which has completely satisfied the laboratory demands which initiated this study.

A.—The circuit illustrated in Fig. 6 represents an ***s*** circuit with a ***u*** circuit arranged to provide the biasing voltage. The left-hand portion of the circuit, involving T_1 and the primed resistors, is exactly like the circuit of Fig. 4(b) and provides a perfectly stabilized voltage across R_3' when R_2'/R_1' is equal to the amplification factor of T_1. The remainder of the circuit is equivalent to that of Fig. 3(b). Since the drop across R_3' is stabilized, the grid of T_2 is effectively connected to the positive input terminal so far as variations are concerned. Thus, the output, or ***s*** portion of the circuit, is in balance when $R_3 = 1/s_{m2}$. Referring to Section 1 above, this is seen to reduce the internal resistance to its minimum value given by $R_0 = (k_{p2} + s_{m2})^{-1}$. A single glow discharge lamp T_3 provides the fixed bias for both portions of the circuit. The most useful feature of this circuit is its adaptability to high voltage, constant-load applications in which the negative side is to be grounded. It should be pointed out again that since s_m varies rapidly with changes in plate current, the condition of balance will be satisfied only for small variations of input voltage. The stabilizer tube T_2 must be rated for the full output voltage.

B.—For high voltage applications allowing the positive side to be grounded we have recommended the ***u*** circuit of Fig. 4(b). In this circuit the series resistance of R_1 and R_2 shunts the rectifier and should be kept high to avoid power loss. However, the entire load current flows through R_1 and this resistance should be kept low to avoid excessive loss of voltage. If these two opposing requirements are met by selecting a high mu tube, the plate resistance and the internal resistance of the stabilizer will be very high. Fig. 7 offers a solution of this dilemma by interposing a stage of amplification between the drop across R_1 and its application between the control grid and cathode of the tube T_2 for whose amplification factor the circuit is balanced. The value of R_1 required for balance may be reduced approximately in the ratio of the voltage gain of the amplifier stage (cf. Table II below) without otherwise altering the design or performance of the basic ***u*** circuit. A low mu tube may therefore be selected for T_2, keeping the internal resistance of the stabilizer low, and the circuit may be designed for as high output voltages as desired so long as the expected *variations* in output voltage do not exceed the voltage rating of T_2. An extra glow discharge lamp, T_4, is required for this circuit in order to secure the proper distribution of d.c. potentials on the grids of the two tubes. A study of the voltage distribution for both extremes of expected input voltage variation must be made in designing the circuit for specified conditions to determine how many lamps should be used in each indicated position.

The precaution of bringing the cathodes to operating temperature before the high voltage is applied should be observed in working with *all* circuits in which the tubes are rated for the *variations* in input voltage. Such procedure will prevent the entire voltage from appearing across the stabilizer tube with probable disastrous results.

C.—The two preceding circuits have been especially adapted for constant-load service. For applications in which the load is variable, a stabilizer of the degenerative type with its low internal resistance is desirable. Fig. 8(a) shows such a circuit in which an ***s*** circuit, as illustrated in Fig. 3(b), is used to provide the fixed bias indicated in the simple circuit of Fig. 5(a). A high internal resistance for the ***s*** portion of the circuit is not material in this case, since no current is drawn from it, so that T_2 may be balanced for a small current and as high a voltage as its rating permits. If T_2 is a high mu transmitting triode (or pentode with proper provision for screen voltage), one 90-volt glow discharge lamp will probably suffice to balance the ***s*** circuit for voltages of the order of one to two thousand. T_1 is then selected according to the stabilization ratio and internal resistance required, and need have a voltage rating only sufficient for the expected variations in input voltage. If a low internal resistance is the chief consideration, T_1 will be selected as a low mu tube with high transconductance. If considerable stabilization against input variations is required, T_1 may be a beam tetrode whose screen polarization is secured by a glow lamp T_3 connected as indicated in Fig. 8(b). This method of obtaining screen voltage for the tetrode is not usually as effective as the use of a separate battery, but it can be used where the battery is undesirable and provides a considerably higher stabilization ratio (for a given internal resistance) than can be secured with triodes.[10] For example, the circuit of Fig. 5(a), employing a type 6V6G tetrode ($u=218$), with a glow lamp used to obtain the screen voltage, had a stabilization ratio of approximately 150 with an internal resistance of 300 ohms. With the circuit of Fig. 8(b) the same performance could be secured at as high a voltage as that for which T_2 is rated. If this circuit is required to maintain a stabilized voltage for *no* external load, enough current must be bled through R_4 to maintain the screen polarizing glow lamp on the flat portion of its voltage characteristic. The bleeder is in shunt with the output and accordingly reduces the internal resistance of the stabilizer at the expense of additional rectifier load.

An interesting possibility arises in connection with the adjustment of this circuit. The foregoing performance was estimated upon the basis of a balance condition for the ***s*** portion of the circuit, resulting in a perfectly stabilized plate-cathode voltage for T_2. If the plate resistor, R_3', of T_2 is made somewhat larger than that required for ***s*** balance, T_2 behaves as an amplifier stage whose plate-cathode voltage varies in phase opposition to the variations in stabilizer input voltage. Such a variable voltage applied to the grid of T_1 may be adjusted to provide perfect stabilization of the voltage across the external load, at the same time preserving the low internal resistance characteristic of the degenerative stabilizer. As might be expected for a circuit capable of being balanced for perfect stabilization, this is equivalent to one of the bridge circuits for the measurement of electron-tube coefficients. In particular, it is the stabilizer analog of the circuit for measurement of negative values of amplification factor, discussed as Fig. 11 in our reference cited.[4] The negative sign of the amplification factor for which the circuit can be balanced is represented in this case by the phase reversal obtained in the amplifier T_2. This circuit is further differentiated from the conventional ***u*** type circuit discussed above in that the external load appears in series with the cathode of T_1, and the negative sides of both input and output circuits are common. The exact conditions of balance for this mode of operation of the circuit are given in Table II below.

D.—When the stabilization and internal resistance offered by the preceding circuit are inadequate, the circuit of Fig. 9 may be used. This circuit represents the addition of a separately powered amplifier stage to the simple degenerative circuit of Fig. 5(b). Because the amplifier stage is introduced between the *output*

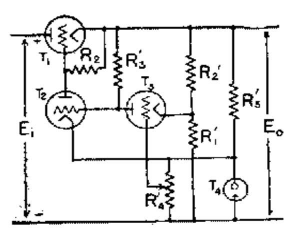


FIG. 10. Degenerative stabilizer with amplified control voltage employing a ***u*** type bridge circuit to provide a stabilized bias voltage. Operation of the latter circuit off balance provides two stages of amplification for the control voltage.

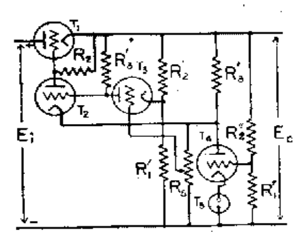


FIG. 11. Degenerative stabilizer with amplified control voltage employing both a ***u*** and an ***s*** type bridge circuit to provide stabilized bias voltages. Operation of the latter circuits off balance provides three stages of amplification for the control voltage.

[10] A modification which is similar, although not employed for screen bias supply, has been described by Bousquet (Electronics, July, 1938) while this paper was in manuscript. It is obvious that the allowable load current is increased over the rating of the stabilizer tube T_1 by the current carried by the shunt circuit—a significant advantage in some cases. The effect of the shunt circuit is to decrease both the stabilization ratio and the internal resistance, the exact expressions being given in Table II.

of the stabilizer and the control tube T_1, instead of between the input and the control tube as in the preceding circuit, the circuit cannot be balanced for perfect stabilization; on the other hand, a very material decrease in the internal resistance is obtained. On account of the phase reversal in the amplifier stage, the fraction of the output voltage between the positive terminal and the cathode tap is applied to the amplifier. If we call this fraction $b(=R_1/(R_1+R_3))$, and the voltage gain of the amplifier stage G_2, the stabilization ratio for the circuit is approximately bG_2u_1, and the internal resistance is $R_0=[k_{p1}+bs_{m1}(1+G_2)]^{-1}$. The performance of the stabilizer is therefore improved as the cathode tap is moved toward the negative output terminal (i.e., as b is increased) but at the expense of requiring a larger bias battery for the grid of the amplifier tube. Instead of increasing b one might choose to increase G_2 by building an amplifier of more than a single stage. In fact, the ultimate limitation of the stabilizer performance (cf. section on stability below) may be made to depend entirely on the stability and gain of the separately powered amplifier. Conventional amplifier design would require an odd number of stages to maintain the proper phase of the control grid voltage of T_1. If tubes with reasonable values of negative transconductance become available, amplifiers with an even number of stages may be used without recourse to the special circuits discussed below.

It is entirely feasible with this circuit to secure a stabilization ratio of 1000 to 2000 with an internal resistance of 10 to 20 ohms. The fraction b is usually made as large as will allow a receiving type amplifier tube to be used. With this provision there is no limit to the voltage for which the circuit may be designed except that the stabilizing tube T_1 must be able to carry the load current and be rated for the expected voltage variations. Professor K. T. Bainbridge has tested a circuit of this type to provide a 2500-volt focusing potential for his mass spectrograph. He reports that for exposures exceeding four minutes the stability of the circuit exceeds that of the bank of B batteries which the stabilizer replaced.

E.—A study of the preceding basic circuits will immediately indicate that a u circuit of the

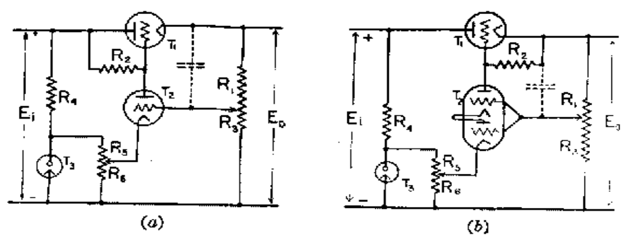


FIG. 12. Two modifications of the degenerative stabilizer with amplified control voltage offering great flexibility in design and excellent performance characteristics.

Fig. 4(b) type could be used to replace the grid biasing battery in Fig. 9. With this addition, and the elimination of the plate battery for the amplifier, the modification of Fig. 10 is obtained. The resistors R_1' and R_2' are adjusted as for the simple u circuit, to stabilize the voltage drop across R_3'. When this condition is satisfied any *change* in E_0 is applied between grid and cathode of the tube T_2 for amplification and control of the stabilizing tube T_1. The performance of the circuit is therefore essentially the same as that given for the circuit of Fig. 9 except that the fraction b is now effectively unity and the batteries have been eliminated. Receiving type tubes may be used at T_2 and T_3 if the output voltage does not exceed 400, but the output voltage may be increased as far as desired by selecting a low power transmitting tube for T_1, having adequate voltage rating. Depending upon the dynamic characteristics of the tubes selected for T_1 and T_2, such a circuit could be expected to exhibit an internal resistance of the order of magnitude of one to ten ohms and a stabilization ratio of the order of 500 to 5000.

Two interesting features appear in connection with the adjustment of this circuit. The performance figures quoted above were predicated upon an adjustment of the u circuit for perfect stabilization of the drop across R_3'. It is possible to adjust the u circuit *off* balance (by decreasing R_2') in such a direction that the drop across R_3' changes in the opposite direction to the voltage change across R_5', thus applying an enhanced differential voltage to the amplifier tube. A practical method of making this transition in the circuit is to replace the variable drop appearing across R_2' by a similar fixed drop across a series of glow discharge lamps. In the limit, this adjustment of the u circuit reduces the function of T_3

to that of a first amplifier stage.[11] The over-all performance of the circuit is calculated by replacing the b of Fig. 9 by the new quantity d which is defined as the ratio of the variational components of e_{g2} and E_0. d is equal to unity if the u circuit is balanced, but may be greater than unity if it is off balance. Under the latter condition it appears to be entirely feasible to secure stabilization ratios of the order of magnitude of 10^4 if certain precautions discussed below under stability are observed.

The second distinguishing feature of the circuit of Fig. 10 lies in the retention of a simple method for varying the output voltage over a wide range. It may be pointed out that the amplification factor of T_3 does not vary markedly for wide variations in its plate current. If a compromise adjustment of R_2' and R_1' is made, such as setting $R_2'/R_1' = u_3/2$, a reasonable degree of amplification will be maintained for wide variations in the steady plate current of T_3. These variations are effected by altering the grid tap indicated on R_4' and produce the desired variation in output voltage by changing the d.c. voltage drop across R_3'. Thus if the rectifier voltage were 3100, and both T_1 and T_2 were rated for this voltage, it would be possible to vary the output voltage from 300 to 3000 by altering the single tap shown on R_4', at all times maintaining a very high stabilization ratio and a very low internal resistance without further adjustment. Alternatively, if the variable tap on R_4' is ganged with a primary control for the rectifier voltage (such as a Variac), only the tube T_2 need be rated for a voltage higher than the expected *variations* in input voltage.

F.—In setting up the preceding circuit of Fig. 10 it is usually desirable to employ two or three glow discharge lamps at T_4 in order to secure a favorable distribution of voltage upon the tubes T_3 and T_2. In the circuit of Fig. 11 the fixed voltage appearing across T_4 of Fig. 10 is replaced by the fixed voltage of an s type stabilizer involving T_4 and the double-primed resistors. Thus only a single discharge lamp, T_5, is required, although the circuit may be arranged to provide as high an output voltage as that for which the amplifier tube T_2 is rated. Most important, however, is the gain in performance of the circuit of Fig. 11 which can be secured by operating the s portion of the circuit off balance in the direction to provide useful amplification of the control voltages. This adjustment is secured by making R_3'' considerably greater than the value required for s balance of T_4. It is to be noted that the grid of T_3 is then no longer held at a fixed potential with respect to ground as is the case in Fig. 10; instead a fraction of the variational off-balance voltage of the s circuit is applied to the grid of T_3. This voltage, amplified by T_3, appears across R_3' even though the resistors R_2' and R_1' are adjusted for u balance. Obviously the net amplification is still further enhanced by adjusting the u tube T_3 off balance also, as in the preceding circuit. Hence the overall performance of the complete stabilizing circuit is that which would be expected from the use of three stages of amplification for the voltage applied to control the series stabilizer tube T_1. Since the three stages of amplification are connected between the output terminals and the series control tube, the internal resistance of the stabilizer is decreased at the same time that the stabilization ratio is increased. The potential performance of this circuit is so great that in considering its departure from perfection (i.e., perfect stabilization and zero internal resistance) it is only necessary in practice to consider such second-order effects as the lack of constancy of voltage drop across the glow discharge lamp and the effect of changes in heater voltage of the first two amplifier stages.

As in the preceding circuit, a simple control of the output voltage over a wide range by variation of a single resistor is retained. Since the tube T_4 is not operated at s balance, a useful voltage gain may be maintained over a wide range of plate current. The output control is effected most readily by making R_1'' a simple rheostat, a decrease in R_1'' being accompanied by an increase in the output voltage and, as before, if R_1'' is ganged with a primary control for the rectifier voltage, only the amplifier tube T_2 need be rated higher than for the expected variations in input voltage.

[11] A stabilizing circuit of this type employing two stages of amplification, but adapted for low voltages and requiring two bias batteries, is described incidentally by Neergaard, Proc. I. R. E. 24, 1207 (1936).

It will be obvious from the sequence of circuit development illustrated in the preceding three sections that one could at this point seek to maximize the gain of the amplifier T_4 by stabilizing the voltage drop across R_2'' with a $\boldsymbol{u}$ type circuit, and that the bias voltage for this circuit could in turn be provided by another $\boldsymbol{s}$ circuit, and that this process could be repeated as far as desirable. It seems unlikely, however, that any further degree of complexity than is represented by Fig. **11** will be justified by the results, and these extensions of the circuit will not be discussed in greater detail.

G.—The last stabilizer configuration to be discussed is that shown in two modifications in Fig. 12(a) and (b). Although simpler than the preceding circuits, it is similar in that it consists of an amplifier stage applied to the basic degenerative circuit. The arrangement is slightly different and its extreme versatility for low voltage (130–600) applications merits further discussion. We developed this circuit to meet our own laboratory demands, but it seems to have appeared independently, and probably earlier, in the RCA laboratories[12] and somewhat later in the Philco and Bell Laboratories.[13]

In the circuit of Fig. 12(a), both the plate supply for the amplifier tube and the glow lamp bleeder are connected to the input side of the stabilizer. This is desirable if in normal service the output voltage is to be varied through wide limits by adjustment of the amplifier control grid tap. Since the rectifier output is essentially constant except for regulation under variable loading, the glow lamp can be kept upon the flat portion of its voltage characteristic for wide variations of output load and for various adjustments of the output voltage. The control grid voltage of T_1 can be brought to zero while still preserving enough drop across R_2 to maintain full gain from the amplifier tube; this insures that the drop across T_1 can be brought to a minimum and the maximum output voltage obtained from a given rectifier. The stabilization ratio for this circuit is given by $S_0=1+u_1(1+aG_{2_r})+r_{p1}/R_i$, where the subscripts refer to the dynamic coefficients of the tubes as numbered, a is the fraction $R_3/(R_3+R_1)$, and G_{2_r} is the effective gain of the amplifier stage. The internal resistance of the stabilizer is given by

$$R_0=\frac{1}{k_{p1}+S_{m1}(1+aG_{2r})}.$$

The circuit modification of Fig. 12(b) is not so well adapted for wide variation of output voltage since the plate voltage available for T_2 is decreased by the drop across T_1. The approximate values of S_0 and R_0 are the same as for the circuit of Fig. 12(a), although the exact expressions indicate that slightly better values may be obtained with this arrangement on account of the greater constancy of the amplifier plate supply voltage.

On the other hand, if a pentode is used for the amplifier T_2, a significant improvement in the stabilization ratio can be obtained by deriving the screen bias voltage from a divider connected across the stabilizer input. In such a case the cathode-screen-plate electrodes of T_2 serve the same function as the off-balance $\boldsymbol{s}$ tube of Fig. 8, and allow the stabilization ratio to be made infinite, or even negative. Since the screen and control grid parameters are not independent, it is awkward to specify the conditions of balance. On the other hand, the "cascode,"[14] illustrated as T_2 in Fig. 12(b), is ideally adapted to perform this function if the control grid of the upper section is returned to a variable tap on R_4. The condition of balance (see Table II) is nearly independent of the position of the variable tap on R_1, R_3, so that the same high degree of stabilization is maintained for all settings of the output voltage. At the same time, the low internal

[12] Beyond the advertising brochures describing the TMV-118 power supply unit, the only RCA publication of this circuit which we have found is a brief nontechnical discussion appearing in RCA Service News, Dec., 1934. See also U. S. Patent No. 2,075,966 issued to A. W. Vance, and RCA Application Note No. 96, Aug. 24, 1938.

[13] Trucksess, Bell Lab. Record **15**, 298 (1937). See also Q. S. T. **21**, 14, Aug. (1937); U. S. Patent No. 2,120,844 issued to R. D. Brown, Jr.

[14] This tube connection we have called the "cascode" and, although somewhat unconventional, it is generally useful for d.c. amplifier work in which it is inconvenient to supply the additional bias voltages for a pentode. The dual triode, as connected, may be shown to be equivalent to a single triode having an amplification factor of $(u_2+u_3+u_2u_3)$ and a plate resistance of $(r_{p3}+(u_3+1)r_{p2})$, where the subscripts 2 and 3 refer to the lower and upper portions of the tube, respectively. These coefficients must be evaluated for the existing voltage distribution between the two sections and experiments indicate that the effective amplification factor for a type 6C8G tube is approximately 1300 with a plate resistance of 3 to 5 megohms.

resistance characteristic of the degenerative stabilizer with amplified control voltage is retained.[15]

It is desirable, from the standpoint of internal resistance (and from the standpoint of stabilization if a triode is used for T_2), to make the fraction a as large as possible. This is conveniently done for rapid fluctuations or audiofrequency variations by adding the bypass condenser shown dotted in Fig. 12; when the reactance of this condenser is negligible with respect to R_3, a is effectively unity. The stabilizer may therefore be several times as effective in suppressing ripple voltages as it is in suppressing slow variations in output. If it is desirable to make a nearly unity for d.c. as well, the voltage drop across R_1 may be replaced by the constant drop obtained from one or more glow lamps.

Either of the foregoing circuits offers very flexible design opportunities to meet specific demands. A typical general purpose unit employs inexpensive "replacement" parts in the rectifier, a single glow lamp of either the 874, 313C, or neon type, a 2A3 stabilizing tube, and either a 6J7 amplifier tube or a 6F8G (or 6C8G) in the cascode connection indicated in Fig. 12(b). Such a unit displays the over-all performance (including the effect of rectifier resistance) indicated by a stabilization ratio of 50–∞, depending on the amplifier connections, and an internal resistance (dynamic, i.e., $a=1$) of one to two ohms. So far as internal resistance is concerned, this performance is roughly equivalent to that offered by a "B" battery composed of lead storage cells of about fifteen ampere-hour capacity. Other designs which have proved useful range from a 50–600 volt, 0.5-ampere unit (using ten 2A3 tubes in parallel), to a variable grid bias unit[16] rated at -250 to $+150$ volts at 50 ma and continuously variable through zero (two stabilizers operating from the same rectifier).

In conclusion it may be pointed out that any of the foregoing circuits which can be balanced for perfect stabilization can also be adjusted to over-compensate slightly, yielding large negative values of S_0. Reference to Eq. (15) will indicate that this feature, combined with practical values of rectifier resistance, makes it possible in some cases to achieve a zero or negative internal resistance. However, when a vanishing internal resistance is secured in this way the stabilization can never, at the same time, be perfect.

Glow Discharge Lamps

In all of the foregoing discussion of stabilizer circuits employing glow discharge lamps as battery substitutes, we have tacitly assumed that the voltage drop across the lamp is constant, and the formulas for stabilization and internal resistance have been computed on this basis. In this section we will discuss the general effect of the failure of this assumption.

The typical voltage-current characteristic of a glow discharge lamp displays a gradual and almost linear rise of voltage as the current through the lamp is increased beyond a characteristic value. Within this region the lamp may be said to have an equivalent circuit composed of an opposed battery whose voltage is that of the (extended) linear intercept on the voltage axis, in series with a resistance equal to the slope of the volt-ampere characteristic. When such a discharge lamp is introduced into an amplifier circuit, as in Fig. 12(a), the variational resistance of the glow lamp must be considered as a degenerative autobiasing resistance which operates to reduce the gain of the amplifier stage. Similarly in the **s** circuit of Fig. 3(b) the presence of a variational resistance in the cathode circuit operates to reduce the effective value of transconductance for which the circuit is to be balanced.

For currents smaller than the characteristic value the slope of the volt-ampere characteristic may become zero and then negative. In some glow lamps this region is rendered unstable by irregular changes in the position and cross section of the glow discharge, while in others it represents a perfectly usable region. This leads to an interesting application in connection with the amplifier stage of stabilizer circuits such as Fig. 12(a).[17] The voltage gain of such an amplifier is given by

$$G_{2_r}=\frac{u_2R_2}{r_{p2}+R_2+r(1+u_2)},$$

[15] The suggested amplifier connections, yielding perfect stabilization, are, of course, applicable also to the circuits of Figs. 10 and 11.

[16] Designed by Mr. J. E. Shepherd, Cruft Laboratory.

[17] We are indebted to Dr. W. N. Tuttle for pointing out this feature.

where r represents the degenerative resistance in series with the cathode, in this case the variational resistance of the glow lamp. It is then possible, as r is made increasingly negative, to secure a very large and, on passing through infinity, a negative amplification with its consequent phase reversal. Thus by appropriate adjustment of the current through the glow lamp one may achieve simultaneously perfect stabilization and a vanishing internal resistance. We have observed this effect but most glow lamps do not appear to be uniform with respect to the negative resistance characteristic. We have therefore adopted the practice of designing the stabilizer unit to have acceptable performance for a positive glow lamp resistance. We then secure what additional performance is possible by individual adjustment of the glow lamp current.

Typical values of variational resistance at selected currents for several types of glow discharge lamp are shown in Table I. These values vary from tube to tube and are presented merely for guidance in stabilizer design.

Stability

Several of the stabilizing circuits which have been discussed indicate the possibility of securing very large stabilization ratios. Since any d.c. stabilizing circuit is somewhat similar to a high gain d.c. amplifier, it will be apparent that spurious d.c. voltages introduced into the circuit from any source will have a serious effect upon the output stability. One such source is the equivalent e.m.f. existing in the grid-cathode circuit of an amplifier tube[18] representing the combined effects of contact potential, initial velocity of emission, and other factors. This e.m.f. may change by as much as 0.1 volt for a ten percent change in heater current. The effect of this upon stabilizer design may be illustrated by a numerical example. Suppose a stabilizer is to deliver 1000±1 volts for ±5 percent line voltage variations. This would normally require a stabilization ratio of 50. However, if the circuit is that of Fig. 12(a) operated with the fraction $a=0.25$, the spurious cathode circuit e.m.f. would itself account for 80 percent of the allowed output variation so that a stabilization ratio of 250 would be required to meet the specifications. Two solutions of the difficulty are available. The heater circuits of the amplifier stages may be supplied from a saturable core type line voltage regulator, or, less expensively, the heater circuit

[18] R. M. Bowie, Proc. I. R. E. **24**, 1501 (1936).

Table I. *Typical values of glow lamp resistance as measured dynamically at a frequency of 1000 c.p.s.*

Type of Lamp	Current Milliamperes	Resistance Ohms	Notes
W.E. 313A	3	620	This type tube displayed the smallest variation from tube to tube. Very stable.
	5	510	
	10	420	
	15	340	
Type 874	5	600	
	10	350	
	15	220	
	20	180	
Type VR150	5	1000	
	10	640	
	20	320	
	30	210	
Type VR90	5	1000	Least reliable and most unstable of all types tested.
Type VR75	10	580	
	20	450	
	30	250	
RCA 991	No resistances less than 1200 ohms were found for currents between 0.004 and 3.0 milliamperes.		
Neon lamp ¼ watt (helical electrode)	0.6	−400	
	2.0	+650	
	5.0	550	
	10.0	350	

TABLE II ~ SUMMARY OF CIRCUIT ANALYSES

FIGURE	CIRCUIT DESIGNATION	VOLTAGE STABILIZATION, S_0	INTERNAL RESISTANCE, R_0	INTERNAL RESISTANCE, R_0 (Alternative Expression)	CONDITIONS OF BALANCE	DEFINITIONS AND REMARKS
3a	S	∞	$\frac{r_p}{1+cu}$	$\frac{1}{k_p+cS_m}$	$S_m=\frac{1}{cR_3}$	$c=\frac{R_1}{R_1+R_2}$ $k_p=\frac{1}{r_p}$
3b	S	∞	Note 2 $\frac{r_p}{\frac{1}{1+r/r_p(1+u)}+cu}$	$\frac{1}{\frac{1}{r_p+r(1+u)}+cS_m}$	$S_m=\frac{1+r/r_p}{cR_3-r}$	r = glow lamp resistance
4a,4b	u	∞	r_p+R_2 Note 2	$\frac{1+S_mR_1}{k_p}$	$u=\frac{R_2}{R_1}$	
5a	D	$1+u+\frac{r_p}{R_L}$	$\frac{r_p}{S_0-r_p/R_L}$	$\frac{1}{k_p+S_m}$		
5b	D	$1+cu+\frac{r_p}{R_L}$	$\frac{r_p}{S_0-r_p/R_L}$	$\frac{1}{k_p+cS_m}$		
6	S+u Bias	∞	$\frac{r_{p_2}}{1+u_2}$	$\frac{1}{k_{p_2}+S_{m_2}}$	$u_1=\frac{R_2'}{R_1'}$ $S_{m_2}=\frac{1}{R_3}$	
7	u+Amplifier	∞	$r_{p_2}+cR_3[1+u_2(1+G_1)]$	Note 2 $r_{p_2}+R_3\left[\frac{1+G_1\left(1-\frac{1}{S_{m_1}R_4}\right)}{1-G_1\left(1-\frac{1+R_2/R_1}{S_{m_1}R_4}\right)}\right]$	$u_2=\frac{R_2}{R_1G_1\left(1+\frac{1}{G_1}-\frac{1+R_2/R_1}{S_{m_1}R_4}\right)}$	$G_1=\frac{u_1\frac{R_4R_5}{R_4+R_5}}{r_{p_1}+\frac{R_4R_5}{R_4+R_5}}$
8a	D+S Bias s circuit balanced	$1+u_1+\frac{r_p}{R_L}$	$\frac{r_{p_1}}{S_0-r_{p_1}/R_L}$	$\frac{1}{k_{p_1}+S_{m_1}}$	$S_{m_2}=\frac{1}{c'R_3'}$	$c'=\frac{R_1'}{R_1'+R_2'}$
8a	D+s = −u u circuit balanced	∞	$\frac{r_{p_1}}{1+u_1}$	$\frac{1}{k_{p_1}+S_{m_1}}$	$u_1=\frac{1}{G_2'\left[c'-\frac{1}{S_{m_2}R_3'}\right]}$	$G_2'=\frac{u_2R_3'}{r_{p_2}+R_3'}$
8b	D+s Bias, Shunted s circuit balanced	$1+\frac{u_1+r_{p_1}/R_L}{1+r_{p_1}/R_S}$	$\frac{r_{p_1}}{S_0(1+r_{p_1}/R_S)-r_{p_1}/R_L}$	$\frac{1}{k_{p_1}+S_{m_1}+k_S}$	$S_{m_2}=\frac{1}{c'R_3'}$	$k_S=\frac{1}{R_S}$
9	D+Amplifier (separately powered)	$1+\frac{r_{p_1}}{R_L}+bG_2u_1\left(1+\frac{1}{G_2}\right)$	$\frac{r_{p_1}}{S_0-r_{p_1}/R_L}$	$\frac{1}{k_{p_1}+bG_2S_{m_1}(1+1/G_2)}$		$b=\frac{R_1}{R_1+R_3}$ $G_2=\frac{u_2R_2}{r_{p_2}+R_2}$
10	D+Amplifier+u	$1+\frac{r_{p_1}}{R_L}+dG_2u_1\left(1+\frac{1}{du_2}\right)\left(1+\frac{1}{S_{m_1}R_2}\right)$	$\frac{r_{p_1}}{S_0-r_{p_1}/R_L}$	$\frac{1}{k_{p_1}+dG_2S_{m_1}(1+1/du_2)(1+1/S_{m_1}R_2)}$	$u_3=\frac{R_2'}{R_1'}$ (But not usually balanced)	$d=1+G_3'\frac{u_3-R_2'/R_1'}{u_3(1+R_2'/R_1')}$ $G_3'=\frac{R_3'u_3}{r_{p_3}+R_3'+(1+u_3)\frac{R_1'R_2'}{R_1'+R_2'}}$
11	D+Amplifier+u+s	$1+\frac{r_{p_1}}{R_L}+(g+f)G_2u_1\left(1+\frac{1+f}{(g+f)u_2}\right)\left(1+\frac{1}{S_{m_1}R_2}\right)$	$\frac{r_{p_1}}{S_0-r_{p_1}/R_L}$	$\frac{1}{k_{p_1}+(g+f)G_2S_{m_1}\left(1+\frac{1+f}{(g+f)u_2}\right)\left(1+\frac{1}{S_{m_1}R_2}\right)}$	$S_{m_4}=\frac{1}{c''R_3''}$ $u_3=\frac{R_2'}{R_1'}$ (But not usually balanced)	n; see Note 1 $c''=\frac{R_1''}{R_1''+R_2''}$ $f=G_4\left(c''-\frac{1}{S_{m_4}R_3''}\right)$ $G_4=\frac{u_4\frac{R_3''R_5''}{R_3''+R_5''}}{r_{p_4}+\frac{R_3''R_5''}{R_3''+R_5''}}$ $G_3''=\frac{R_3''u_3}{r_{p_3}+R_3''+(1+u_3)\frac{R_1''R_2''}{R_1''+R_2''}}$ $g=1+G_3''\left[nf+\frac{u_3-R_2'/R_1'}{u_3(1+R_2'/R_1')}\right]$
12a	D+Amplifier	$1+\frac{r_{p_1}}{R_L}+aG_{2r}u_1\left(1+\frac{1}{aG_{2r}}\right)$	$\frac{r_{p_1}}{S_0-r_{p_1}/R_L}$	$\frac{1}{k_{p_1}+aG_{2r}S_{m_1}(1+1/aG_{2r})}$		$a=\frac{R_3}{R_1+R_3}$ $G_{2r}=\frac{u_2R_2}{r_{p_2}+R_2+r(1+u_2)}$
12b	D+Amplifier	$1+\frac{r_{p_1}}{R_L}+aG_5u_1\left(1+\frac{1}{au_c}\right)\left(1+\frac{1}{S_{m_1}R_2}\right)$	$\frac{r_{p_1}}{S_0-r_{p_1}/R_L}$	$\frac{1}{k_{p_1}+aG_5S_{m_1}(1+1/au_c)(1+1/S_{m_1}R_2)}$		$u_c=u_2+u_3+u_2u_3$ $r_{p_c}=r_{p_2}(1+u_3)+r_{p_3}$ $G_5=\frac{u_cR_2}{r_{p_c}+R_2}$
12b	D+s+Amplifier = −u+Amplifier	∞	$\frac{r_{p_1}}{1+apu_c\left[1+\frac{1}{u_3}\left(1+\frac{1}{au_c}\right)\right]}$ Note 2	$\frac{1}{k_{p_1}+aG_6S_{m_1}\left[1+\frac{1}{u_3}\left(1+\frac{1}{au_c}\right)\right]\left[1+\frac{1}{S_{m_1}R_2}\right]}$	$u_1=\frac{u_2}{pG_6\left(1+\frac{1}{S_{m_1}R_2}\right)}$	$G_6=\frac{u_2u_3R_2}{r_{p_3}+R_2+r_{p_2}(1+u_3)}$ p; see Note 1

Note 1. The quantities a, b, d, f, g, n, p, which appear in the table, are defined as follows: a = fraction of variational output voltage appearing between ground and a voltage divider tap. b = fraction of variational output voltage appearing between positive terminal and a voltage divider tap. d = fraction of variational output voltage appearing between grid of T_3 and ground. f = fraction of variational output voltage appearing between cathode of T_2 (negative) and ground. g = fraction of variational output voltage appearing between grid of T_2 (positive) and ground. n = fraction of R_5 between tap for T_3 grid and ground. p = fraction of variational input voltage E_i between the upper grid of the cascode and ground.

Note 2. This expression has been derived assuming the condition of balance to be satisfied